Banks vs Zombies

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Abstract

Banks have been observed to lend at subsidized rates to cover the losses of insolvent borrowers or "zombie firms." I study the incentives to restructure debt in a borrower-lender game with risk shifting under debt overhang. I provide conditions under which it is optimal to engage in zombie-lending even when socially inefficient. The main driver is that when a creditor becomes insolvent, the firm looses access to the competitive funds market and instead the incumbent bank has monopoly power. The bank can either liquidate the firm, fund investment or zombie-lend. Liquidation is not desirable given that disruption costs destroy value. Funding investment requires a large repayment which incentivizes firms to take more risk, thus decreasing the overall value for the bank. Thus, the bank prefers to zombie-lend taking as much as possible from the firm's revenue. When the lender is also financially distressed, the bank may keep the firm afloat to prevent the risk of its own bankruptcy. Zombie-lending can happen for profitable investments and allowing for renegotiation does not solve the problem. I discuss policy alternatives and show that monetary policy and bank capitalization can be ineffective, and that debt haircuts are necessary.

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1 Introduction

Movements in asset prices can destroy collateral values and spell trouble for both banks and firms. Insolvent firms may continue to operate if banks roll over their debt at subsidized rates. These "zombie firms" survive but have a hard time growing or investing in new projects. This phenomenon arose in Japan in the early 90s and led to widespread misallocation of resources, and a lower productivity of the economy [Caballero et al., 2008]. Debt renegotiation should, in principle, lead to both parties reaching a welfare improving situation in the face of debt overhang. However, the experience shows that this is not necessarily the case.

Zombie lending is not exclusive to Japan. The phenomenon has also been observed in Europe. Several features are common to these crises. In particular, the number of zombie firms spikes after the burst of an asset bubble, and they are often in a lending relationship with a poorly capitalized bank. Moreover, these firms are less likely to invest. From a policy side, bank recapitalization and lowering the interest rate proved unsuccessful in igniting growth (see Section 2).

I analyze the incentives of lenders and borrowers to explain why zombie-lending continues to thrive and is not solved by the market. I consider a bank-firm game with two main ingredients: risk shifting and debt overhang. In this setup, I find a set of conditions under which zombie lending arises as an equilibrium outcome of the renegotiation process. The implications of the model are aligned with stylized facts. I use the model to extract policy implications regarding monetary policy, debt haircuts and capital injections.

The main mechanism of the model is that a firm that suddenly becomes insolvent looses access to the competitive credit market, becoming locked in with its incumbent bank. This creates a disconnection between the incentives of the bank and the firm. The bank can extract a larger surplus from the captive firm. On the other hand, overburdened firms prefer not to use the funds efficiently and to instead increase the risk in their investment project. Anticipating the firm's behavior, the incumbent bank will not extend fresh funds. Moreover, the bank will decide not to liquidate the firm because the disruption costs are large or because such a move would force it to acknowledge losses in its balance sheet and increase the risk of becoming insolvent itself as well.

The model consists of three agents: a firm and two banks that play for three periods. The firm starts with an exogenous level of debt contracted with one bank, the incumbent. The firm has access to a continuum of projects differing in risk. In normal times, when initial debt is low, the firm can repay its incumbent bank, and borrow from any of the two banks to finance investment. Bank competition pushes the cost of borrowing until banks make zero profits. The firm invests in a project with socially optimal risk and social welfare is maximized.

In crisis times, the firm becomes insolvent and banks may become distressed by a shock. I model the crisis as a shock of zero measure and assume that firms will never become solvent at any point in the future. If it were not the case, firms could attract funds from other investors. Previous explanations for delays in restructuring rely on asymmetric information or uncertainty [Vilanova, 2004]. In my model, shocks are permanent and there is no uncertainty on the future evolution of the firm's average output. Following from the assumption that old debt is senior to new debt, a hold-up problem arises. The firm is no longer able to find funds in the competitive market, and the incumbent bank becomes a monopolist.

As a monopolist, the bank has two broad strategies in hand: it can liquidate the firm or extend credit. The firm, in turn, can use the funds with discretion. At the end of the game, the bank is either repaid or it liquidates the firm. In this model, I show that there are various situations under which the bank decides to keep the firm in the market, even if insolvent. In other words, the bank zombie-lends. Importantly, this can happen even if the firm has a profitable project.

The bank will decide not to extend enough funds for the investment project when there is large scope for risk shifting. This is because any funds extended to a distressed firm will end up being "gambled" with low expected returns for the bank. If the bank liquidates the firm, it must pay disruption costs. In response, the bank prefers to take the business-as-usual revenue stream for itself.

I extend the model to analyze the case of a distressed bank and firm simultaneously. I model this in a setting where a bank may suffer from a shock to its assets. If the bank liquidates the firm in period 0, it faces a shock in period 1 that may render itself bankrupt. Liquidating a firm implies recognizing lost assets. A bank in a strong capital condition would liquidate the firm and acknowledge the loss it in its books. However, a weak bank may not be willing to do so, since it may increase its own bankruptcy risk.

The model yields several policy prescriptions. First, a haircut on firm debt can increase welfare if banks are in a strong position. A firm in debt overhang is overburdened by debt, and thus decides to increase the risk of its project. A haircut on debt decreases the bank's market power, allowing the firm to obtain funds and invest. Social surplus becomes larger and can be used for a welfare-improving redistribution such that the bank is also better off. Second, recapitalization may be either ineffective or induce the inefficient liquidation of the firm. It will not, however, increase investment. Recapitalizing a bank without a haircut on corporate debt does not change the firm's incentives and is not enough to resume investment. The optimal policy response to zombie-lending includes both bank recapitalization and a debt haircut. Third, when risk shifting is prevalent and disruption costs particularly large, monetary policy is ineffective. Decreasing the interest rate decreases the cost of investment, but does not affect the underlying incentive problems.

Related Literature. Several papers dealt empirically with the presence of zombie-lending in Japan since the early 90s. Caballero et al. [2008] complemented this literature with a theoretical model of the effects of zombie firms on the rest of the economy. My paper relates to this literature by explaining how this phenomenon can arise in a decentralized equilibrium. As such, it is linked to the literature on Debt Forbearance. Hellwig et al. [2012] distinguish different types of forbearance: among the supervisor and the bank, and among the bank and the debtor. I abstract from the incentives of the regulators to solely focus on profit-seeking agents. Berglof and Roland [1998] was the first paper to study troubled firms that fail to exit the market. However, their focus is on state owned companies, and a government that wants to avoid social costs. I, instead, show how competitive markets can lead to the same situation.

To build this model, I combine insights from three bodies of literature: risk shifting, debt overhang, and renegotiation.

Myers [1977] shows that debt overhang leads to under-investment by firms. This is due to the difficulties in raising capital for new investment, given that the profits would benefit existing debt holders first, instead of the new investors. This spanned a large literature.[1] Kovrijnykh and Szentes [2007] study sovereign countries in debt overhang. Their setup is very closely related to my paper. In particular, I model market structure in a very similar fashion. My model can be considered a simplified version of Kovrijnykh and Szentes [2007] that focuses on firms. Unlike countries, firms can be liquidated and have large scope for risk shifting. These two differences substantially change the space of strategies and

[1] For example, Hennessy [2004], Titman and Tsyplakov [2007], Moyen [2007], Diamond and Rajan [2009], and Occhino and Pescatori [2015].

outcomes. The incentives of financially distressed firms to gamble with their assets. or risk shifting, was first in Jensen and Meckling [1976]. The evidence is mixed,[2] and the literature has explored theoretically several mitigating factors.[3] My proposed mechanism can account for why the mechanism is elusive in the data.

Debt renegotiation has received substantial attention since the seminal papers of Hart and Moore [1998], Hellwig [1977], and Hart and Tirole [1988]. Restructuring generates a bargaining surplus for both parties and default is inefficient. I focus on a case where the bank acquires all the bargaining power, arising from the debt seniority assumption as in Myers [1977].

Several papers combine insights from one or more of these bodies. Favara et al. [2017] mix the underinvestment result from Myers [1977] with renegotiation. They find that the prospect of easier renegotiations mitigates the underinvestment result. I, on the contrary, focus on banks' opportunistic behavior. Vilanova [2004] develop a model of restructuring for firms in distress, where banks have an option value of waiting for changes in firms' fundamentals. They focus on the banks' opportunistic behavior but, unlike my paper, they do not account for moral hazard by the firms. Moreover, in my model, there is no uncertainty regarding the firm's expected recovery.

Manso [2008] presents a model of risk shifting, in which debt distorts new projects and analyze the effect of allowing for default. This paper does not consider the incentives of the lender and debt restructuring, which are the focus of my paper.

Frantz and Instefjord [2019] combine debt overhang and debt restructuring. Debt restructuring mitigates underinvestment, and is aimed at managing the borrowing policy in non-distressed states. I include risk shifting in a simpler model, which substantially changes results.

Pawlina [2010] shows that shareholders' option to renegotiate debt exacerbates the debt overhang underinvestment result due to the higher wealth transfer after investment. Efficiency is restored by granting creditors the entire bargaining power and the availability of efficient projects alleviates the problem. My results are in stark contrast with this result.

Lastly, a final salient feature of the Japanese and European crises is that, despite

[2] See for example, De Jong and Van Dijk [2007], Eisdorfer [2008], and Gilje [2016].
[3] See for example, Almeida et al. [2011] and Barnea et al. [1980].

lower interest rates and bank recapitalization, investment did not reignite. There is a rich discussion about the desirability of lowering the interest rate in this context.[4] Moreover, current explanations on why banks hoard cash instead of lending it rely on uncertainty. I offer an alternative explanation for the lack of reaction of investment and for cash hoarding based on risk shifting by financially distressed firms.

The paper is structured as follows. Section 2 summarizes results of several empirical papers in the form of stylized facts that motivate this paper. Section 3 presents the model and Section 4 analyzes policy implications. Section 5 discusses the results and assumptions, and concludes.

2 Stylized Facts

Zombie firms were first identified as such in Japan after the burst of an asset and real state bubble. Japan's growth remained stagnant for decades since. Several papers have documented similar features for Europe during the recent crisis. In this section, I highlight a number of stylized facts common to both cases, that motivate and guide my theory.

1. *Zombie firms become more widespread after the burst of a bubble.* Caballero et al. [2008] identify zombie firms in Japan as those receiving a subsidized rate in their loans. They found zombies to be prevalent, and that its number substantially increased after the collapse of the bubble (Figure 1). Aragon [2018] finds the same pattern for Spain.

2. *Highly indebted firms have a higher likelihood of being associated with a financially distressed bank.* Peek and Rosengren [2005] find that undercapitalized banks are more likely to lend to poorly performing firms in Japan. Andrews and Petroulakis [2019] present cross country evidence for European countries of this relationship. This is particularly apparent in countries in which insolvency procedures inhibit corporate restructuring.

3. *Zombie firms or highly indebted firms are less likely to invest.* This is a classic debt overhang result with a large empirical literature supporting it. For example, Peek and Rosengren [2005] and Kalemli-Ozcan et al. [2018].

[4] See Krugman et al. [1998], and Obstfeld and Duval [2018]. among others.

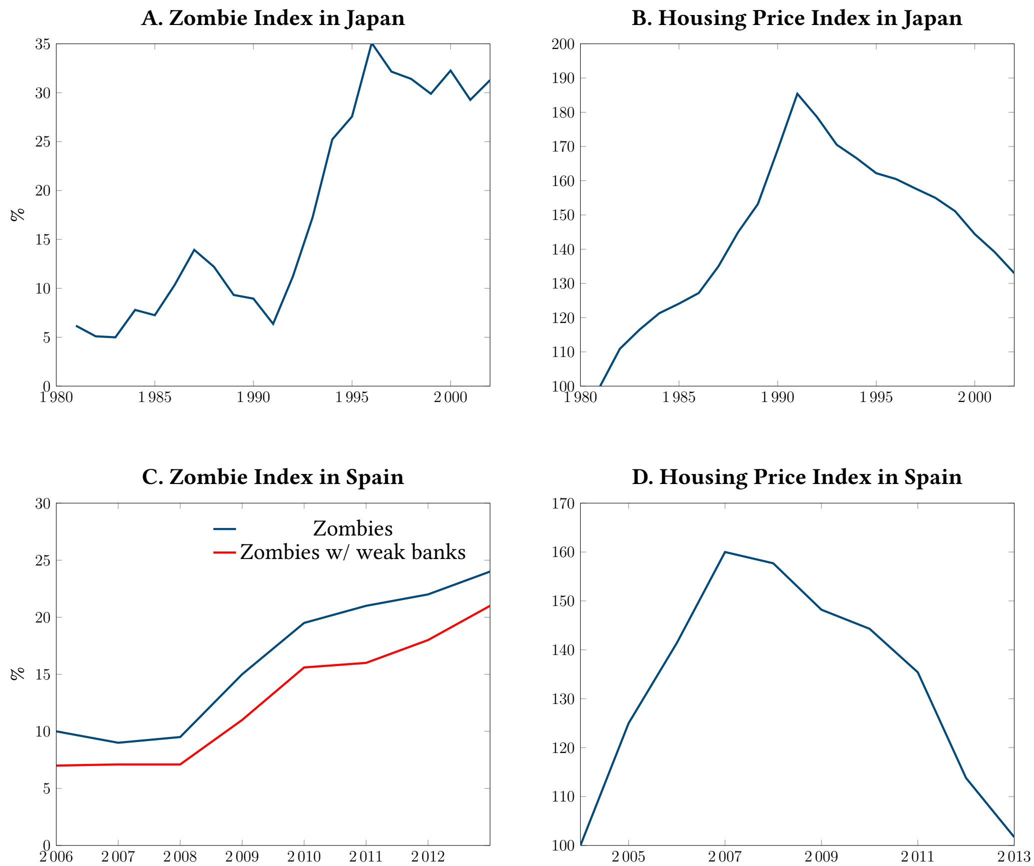

Figure 1: Percentage of Zombie Firms and Housing Price Index in Japan and Spain. Source: Aragon [2018] and Caballero et al. [2008].

4. *Zombie firms have negative effects on the economy.* Several channels were explored empirically: slowdown in productivity [Andrews and Petroulakis, 2019], misallocation of resources Gopinath et al. [2017], declining business dynamics [Decker et al., 2018]. Caballero et al. [2008] and Ahearne and Shinada [2005] show that zombie firms had negative effects on job creation, as well as on non-zombie firms' productivity.

5. *Bank recapitalizations had mild effects.* Acharya et al. [2016] analyze the real effects of the Outright Monetary Transactions Program, which increased the price of European sovereign debt, thus recapitalizing banks.

They find that banks that were recapitalized did increase their lending but they directed it to troubled firms, that failed to invest. Schivardi et al. [2017] and Giannetti and Simonov [2013] present similar results.

6. *Despite lower interest rates, investment did not increase* Interest rates have remained close to zero for both Europe and Japan, but investment did not resume [Krugman et al., 1998] [Eggertsson et al., 2014].

3 The Model

In this section, I present a simple game that provides theoretical insights on the behavior of borrowers and lenders in a situation of debt overhang when risk shifting is allowed. I show that once there is a situation of debt overhang, the decentralized renegotiation equilibrium may be inefficient. I show that an equilibrium exists where firms fail to invest in available profitable opportunities even though banks could finance them. I also analyze different policies and show how they can implement an efficient allocation.

3.1 Environment

There are three risk neutral agents: a firm and two banks. They live for three periods: 0, 1, 2. The firm seeks resources to finance a risky project in a lending market and has an inherited debt with one of the banks, the incumbent. In period 0, banks compete *á la* Bertrand to provide funds to the firm.

The firm possess a technology to produce output. In period 1, the firm produces using its traditional technology. If it has enough funds, it may also carry on an investment project. The investment project, if successful, changes the technology of the firm, which is used in period 2 to produce and pay back debt. If the investment project fails, the firm is liquidated. The timing of the game is in Figure 2.

Banks. There are two banks. The incumbent bank and the competitor bank. The incumbent bank has a claim on firm debt D_0. In period 0, each bank can offer two types of contracts: flowing funds or liquidation.

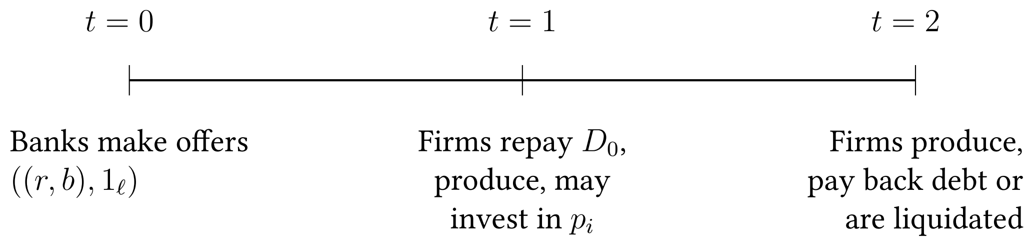

Figure 2: Timing of the model

If the bank decides not to liquidate, it offers a pair (r, b) composed by an immediate payment r and a payment in the next period, b. Note that r can be either greater than zero (in which the bank charges an amount) or smaller than zero (in which the bank extends funds). The bank can also decide to liquidate the firm. The bank has an opportunity cost $C(x)$ for any funds x.

The incumbent bank's expected profits, $E\Pi$, of offering a contract $((r, b), \mathbb{1}_\ell)$, where $\mathbb{1}_\ell$ is an indicator function equal to one if the bank liquidates and zero otherwise, are given by

$$E\Pi(\mathbb{1}_\ell, (r, b)) = (1 - \mathbb{1}_\ell)\left\{p(b)b + (1 - p(b))\ell - C(r) + D_0\right\} + \mathbb{1}_\ell \ell \tag{1}$$

conditional on the contract being accepted by the firm. In Equation 1, $p(b)$ is the success probability of the firm's project, and ℓ is the liquidation value of the firm for the bank if the firm is unsuccessful. The liquidation value of the firm is $\ell = \phi E\pi$, where $E\pi$ is the value of the firm in period 1, and $1 - \phi$ are the disruption costs. Similarly, the competitor bank's profits, $E\Pi'$ of offering contract $((r', b'), \mathbb{1}'_\ell)$ are

$$E\Pi'(\mathbb{1}'_\ell, (r', b')) = (1 - \mathbb{1}'_\ell)\left\{p(b')b' + (1 - p(b'))\ell - C(r')\right\} + \mathbb{1}'_\ell \ell \tag{2}$$

also conditional on the contract being accepted.

Three remarks are in order. First, the fact that the bank has the firm's probability of success as a function of repayment, $p(b)$, shows that it is internalizing the opportunities for risk shifting of the firm. Second, even though they will face different incentive constraints, the only difference between the profit schedules of the banks is that D_0 only appears in the incumbent bank's profit function. Finally, notice that the competitor bank can offer to liquidate the firm, but this is an offer not chosen in equilibrium.

Firms. The firm has a technology that allows it to obtain a revenue of $y_0 = p_0^{-\alpha}$, in case of success that happens with probability p_0. If the project is unsuccessful, the firm gets zero. The expected revenue will remain $p_0 y_0$ without new investment. In other words, this is the "business as usual" revenue stream.

In period 1, the firm can invest in a risky project that improves technology. Projects are chosen from a continuum and are indexed by their probability of success $p_i \in [0, 1]$. In case of success, each project induces an increase in revenue from $y_0 \equiv p_0^{-\alpha}$ to $p_i^{-\alpha}$. The parameter $\alpha \in [0, 1]$ captures the elasticity of revenue to the riskiness of the project. In this way, it is a measure of risk shifting: the safer is the project, the lower is the revenue in case of success. I assume that the firm can only carry on one investment project.

The firm also has an exogenous, previously contracted with a bank, level of debt, D_0 that needs to be paid in period 1. If the firm refuses to pay the contracted debt, it can be liquidated and its assets seized. The cost of investment, X, is the same for all projects p_i. Finally, I assume that the choice of project is not contractible: the firm borrows from the bank but cannot commit to invest following a particular risk profile.

The value of the firm at the beginning of the game is,

$$\max_{\mathbb{1}_I} E\pi = p_0 y_0 - r - \mathbb{1}_I X - D_0 + \max\{\mathbb{1}_I EV_i(b), (1 - \mathbb{1}_I)(p_0 y_0 - b)\} \quad (3)$$

Where y_0 is the level of output, $\mathbb{1}_I$ is an indicator function equal to one if the firm invests, and X is the cost of investment. r is the immediate repayment to the bank on new debt, which may be positive (in which the bank takes funds) or negative (in which the bank extends funds). Notice that r may be contracted with a different bank than the one that lent D_0. The value of the firm in period 2, if it decides to carry on project i, is

$$EV_i = \max_i \left\{ p_i (p_i^{-\alpha} - b) \right\}$$

where p_i is the probability of success of project i, b is repayment on new funds. The firm can choose to carry on a new project or remain in their business as usual, obtaining an expected revenue of $p_0 y_0$. Note that EV_i has an inverse-U shape with a maximum at an intermediate efficient level of risk taking. By taking first order conditions, we can see that, If the firms decides to invest, the project chosen is

$$p^*(b) = \left(\frac{1-\alpha}{b}\right)^{1/\alpha} \tag{4}$$

which depends on b. Equation 4 shows how the firm's attitudes towards risk are shaped by the level of repayment expected by its bank. The reaction function presents risk shifting: the higher the debt, the lower the probability of success of the project chosen in equilibrium.

Incentive Constraints. The problem for each agent is to maximize their profit functions as stated in the previous section. However, there are several incentive constraints that should be taken into account when banks make offers and firms decide whether to accept or reject such offers.

First, the firm will accept any contract with weakly positive profits since otherwise it would be liquidated (in which case the firm gets zero).

(A) IC for firm

$$(1-\mathbb{1}_\ell)\left\{p_0 y_0 - r - \mathbb{1}_I X + \max_i\{\mathbb{1}_I EV_i(b), (1-\mathbb{1}_I)(p_0 y_0 - b)\} - D_0\right\} \geq 0 \tag{5}$$

In words, if the firm is not liquidated, it will only operate if profits are weakly positive, whether investing or keeping its business-as-usual revenue, $p_0 y_0$.

Second, following Myers [1977] and Fama and Miller [1972], I assume debt seniority in repayment.

Assumption 3.1. *Old debt is senior to new debt*

Assumption 3.1, is crucial for the results of the paper. The reasoning is that if it were not the case, when new claims are issued, it would decrease expected repayment for the incumbent bank. If the firm becomes insolvent, the incumbent bank would have to share the recovery value with the competitor. Anticipating this, all banks would refuse to lend in the first place. As Kovrijnykh and Szentes [2007] show, this is equivalent to say that the competitor bank will make an offer

(r', b') only if the firm can make enough profits as to pay its original debt, D_0. The following constraint describes these contracts,

(B) IC for Competitor Bank

$$\Big\{p_0 y_0 - r' - \mathbb{1}_I X + \max_i \{\mathbb{1}_I EV_i(b'), (1 - \mathbb{1}_I)(p_0 y_0 - b)\}\Big\} \geq D_0 \qquad (6)$$

On the other hand, the incumbent bank offers a contract $\{(r, b), \mathbb{1}_\ell\}$ such that the expected repayment from the firm is at least as high as the opportunity cost of all the extended funds in the first period. If the competitor offers a contract that satisfies this, it is also profitable for the incumbent bank to do the same. Without loss of generality, I assume that the incumbent is the one accepted when both banks make the same offer. Therefore, the following incentive compatibility will only apply in equilibrium to the incumbent bank,

(C) IC for Incumbent Bank

$$(1 - \mathbb{1}_\ell)\,\{p(b)b + (1 - p(b))\ell\} + \mathbb{1}_\ell \ell \geq (1 - \mathbb{1}_\ell)\,C(r)$$

where $p(b)$ is success of the project and ℓ is the liquidation value of the firm in case of failure.

Lastly, the project needs to be feasible, in the sense that profits cannot be negative,

(D) Feasibility

$$y_0 - r - \mathbb{1}_I X \geq 0 \qquad (7)$$

3.2 Equilibrium Analysis: Market Structure

Market structure depends on the inherited level of debt. For low levels of debt, the firm is solvent and the competitor bank is willing to make offers that push the incumbent to the competitive outcome. Both banks will make a competitive offer to finance investment. The incumbent bank will make an additional offer in which it requests the original debt to be repaid. On the other hand, if the inherited level of debt is large enough for the firm to be insolvent, then the incumbent bank becomes a monopolist. This is a situation of debt overhang, and the incumbent bank will maximize profits subject to the incentive constraints of the firm, in which it will choose investment from its full array of projects. This is summarized in Proposition 3.2.

Proposition 3.2. (Market Structure) *There exists a threshold for the inherited level of debt, $\bar{D}(y_0)$, such that*

- *(competitive markets) For $D_0 < \bar{D}(y_0)$, if the project is efficient, both banks make offer $\hat{r} = -X$ and $\hat{b}$ equal to the opportunity cost of funds, $\hat{b} = C(\hat{r})$. The incumbent bank makes an additional offer of $\hat{\hat{r}} = D_0$.*
- *(debt overhang) For $D_0 \geq \bar{D}(y_0) > y_0$, the incumbent bank maximizes Equation (1), subject to Equation (7) and Equation (4) if it extends new funds.*

The threshold for debt is a function on initial output, as the relevant measure is the debt to output ratio. In other words, to generate debt overhang, it is equivalent to shock the original level of output (decreasing it) or the original level of debt (increasing it). It is worth noting that I do not explain how debt arrives to that level. Kovrijnykh and Szentes [2007] endogeneize this process as a sequence of bad shocks. In their model, debt overhang happens with probability one, but the agent takes precautions to decrease its occurrence. To isolate this effect, in Section 3.4, I will simply assume that this level is hit by a zero measure shock that pushes original debt beyond the threshold.

I will make the following simplifying assumptions,

Assumption 3.3. *(simplifying assumptions)*

1. *The opportunity cost is given by $C(x) = \hat{p}Rx + (1 - \hat{p})\hat{\ell}$ with return R in case of success with probability $\hat{p}$, and a return $\hat{\ell}$ in case of failure; such that $\hat{p} = p(\hat{b})$ and $\hat{\ell} = \ell$.*
2. *The original technology of the firm, $y_0 = p_0^{-\alpha}$, with $p_0 = \hat{p}$.*

The first assumption about the shape of the opportunity cost function simply makes the bank's opportunity cost risky. This investment pays a return R for each unit of funds if successful, with probability $\hat{p}$, and a pays value of $\hat{\ell}$ if unsuccessful. To simplify calculations, the assumption makes this value equal to the competitive probability of success and the value in case of failure equal to the value of the firm. Including risk in the alternative investment is purely for simplifying the algebra, but results hold with a risk-free investment opportunity.

Throughout the paper, I will focus on investment projects that are efficient. The following assumption will become apparent afterwards, as the bounds ensure that the probabilities of success of projects are bounded by one and projects are efficient. Efficiency in this paper is defined such that these projects are carried on under competitive markets.

Assumption 3.4. *The cost of investment satisfies*

$$RX \in I_X \equiv \left[(1-\alpha), (1-\alpha)\left(\frac{\alpha}{y_0}\right)^{\alpha/(1-\alpha)}\right]$$

with $R \geq 1$.

In the next sections I will analyze the properties of the equilibrium under both competitive and non competitive markets.

3.3 Equilibrium under Competition

When the firm has access to competitive lending, competition will push banks' profits towards zero. Both banks will offer $b = \hat{b}$, offer to extend funds $r = -X$, and the incumbent bank will also get D_0. In this section, I normalize D_0 to zero purely for easiness of exposition, as it does not distort any decision. I assume without loss of generality that the firm will choose its incumbent bank when presented with equal offers from both. The following proposition summarizes the reaction of the firm under competitive markets.

Proposition 3.5. *Under competitive markets as in Proposition 3.2, if Assumption 3.3 holds,* $RX \in I_X$, *and* $D_0 \leq \bar{D}(y_0)$, *then the firm will do project* $\hat{p}$, *given by*

$$\hat{p} = p(\hat{b}) = \left(\frac{1-\alpha}{RX}\right)^{1/\alpha}$$

and welfare is maximized.

In other words, risk neutral banks will compete in prices providing loans until they make an offer at the opportunity cost of those funds. The firm will carry on the project given that it is preferable to the revenue stream from its business-as-usual technology. In doing so, it will take on an efficient level of risk, given by the reaction function of the firm. Firms repay their debt (normalized to zero), and request funds to do an investment project. They receive the competitive rate and take on an optimal level of risk.

3.4 Equilibrium under Monopolistic Banks

Now suppose there is a zero measure shock to original debt, ξ, such that $D_0' = D_0 + \xi \geq \bar{D}$. Following Proposition 3.2, the bank will have monopoly power given that the firm is now insolvent. Modelling debt overhang in this way allows me to abstract from ex-ante precautionary motives. Note that a shock to $\bar{D}$ is equivalent to a shock to y_0, such that $y_0' = \epsilon y_0 = \epsilon p_0^{-\alpha}$ with $\epsilon[0, 1]$. I split this in order to analyze the effects with respect to the size of the drop in output.

According to Proposition 3.2, bank now faces a maximization problem subject to incentives constraints. In this case, it can act as a monopolist and extract more than the competitive rate. The bank has two possible strategies, to offer either (r, b) or to liquidate the firm. It is important to make two remarks. First, initial debt D_0' is a sunk cost and will play no role beyond granting monopoly power. Second, even though (r, b) is a continuous decision, there will only be two cases to consider for the initial repayment: (a) $r^Z = \epsilon p_0 y_0$ and (b) $r^I = \epsilon p_0 y_0 - X$. Option (a) is defined as zombie lending, and option (b) is the case in which the bank still extends funds to the firm for investment. In both cases, the firm gets zero profits in period 1. Any intermediate case is not an equilibrium because there are either idle funds or not enough to invest. This is formally stated in Proposition 3.6.

Proposition 3.6. *Under debt overhang as in Proposition 3.2, the set of bank's strategies, σ, is dominated by one of the following strategies,*

1. *(I) Funding investment, in which case it sets $r^I = \epsilon p_0 y_0 - X, b^I = \frac{\ell}{1-\alpha}$,*
2. *(Z) Zombie-lending, in which case it sets $r^Z = \epsilon p_0 y_0, b^Z = \epsilon p_0 y_0$*
3. *(ℓ) Liquidating, in which case it sets $\mathbb{1}_\ell = 1$.*

In Figure 3 the profits for each of the equilibrium strategies are shown. Funding investment via (r^I, b^I) gives the bank $E\pi^I$, which is hump-shaped as the increase in b increases the risk chosen by the firm. An increase in α, meaning higher scope for risk shifting, flattens the curve as shown in the dotted line. Liquidation ($\mathbb{1}_\ell = 1$) gives the bank $E\Pi^\ell$ which is a constant. If the bank prefers to offer (r^Z, b^Z), the firm does not change its technology and thus there is no scope for risk shifting (and, as such, probability of success remains p_0). Thus, profits increase monotonically until they are truncated due to feasibility. Formally, the bank solves the following maximization problem,

$$\max_{\sigma\in\{Z,I,\ell\}} \left\{E\Pi^I(r^I, b^I), E\Pi^Z(r^Z, b^Z), E\Pi^\ell(\mathbb{1}_\ell = 1)\right\}$$

$$\begin{aligned}
&\text{subject to} \\
(i) \quad & \epsilon y_0 - X \geq r \\
(ii) \quad & p(b) = \left(\frac{1-\alpha}{b}\right)^{1/\alpha} \quad \text{if} \quad \sigma = I
\end{aligned}$$

where

$$\begin{aligned}
E\Pi^I(r^I, b^I) \quad &= p(b^I)b^I + (1 - p(b^I))\ell + C(r^I) + D_0 \\
E\Pi^Z(r^Z, b^Z) \quad &= p_0 b^Z + (1 - p_0)\ell + C(r^Z) + D_0 \\
E\Pi^\ell(\mathbb{1}_\ell = 1) \quad &= C(\ell)
\end{aligned}$$

When there is debt overhang problem, the amount of money that the bank can obtain from the firm is not determined by the opportunity cost, but it maximizes the amount of money that can be extracted from a firm. Therefore, the bank can push the debt beyond the competitive level of debt. The higher pressure pushes the firm towards higher risk taking thus lowering social welfare. The following result follows from this proposition,

Lemma 3.7. *When firms are in debt overhang as in Proposition 3.2, $RX \in I_X$, and $\ell \geq (1-\alpha)^2$, social welfare is not higher than under competitive markets. Liquidation is socially inefficient.*

This Lemma states that, under debt overhang, social welfare is lower. The restriction of $\ell \geq (1-\alpha)^2$ simply ensures that probability is well defined under b^I. Welfare is lower due to an unnecessary level of risk taking, unexploited investment opportunities or inefficient liquidation.

A zombie firm is a firm with profitable projects that is not investing and is not solvent. Lack of solvency is ensured by the definition of debt overhang. I now find conditions under which the zombie-lending strategy will be chosen by the bank, even when there are profitable projects. This is the main result of the paper.

Theorem 3.8. *(Zombie Firms) Let $\Omega(\epsilon, \phi, \alpha, RX) \in [0,1]^4 \times I_X$ be the set of admissible parameters. Then, under debt overhang as in Proposition 3.2, when $\alpha \to 1$, when*

$$\frac{\epsilon}{1+\epsilon} \geq \phi \frac{1}{1+X}\left(1 - X + \frac{1}{R}\right) \tag{8}$$

then $E\Pi^Z \geq E\Pi^I$ *and* $E\Pi^Z \geq E\Pi^\ell$.

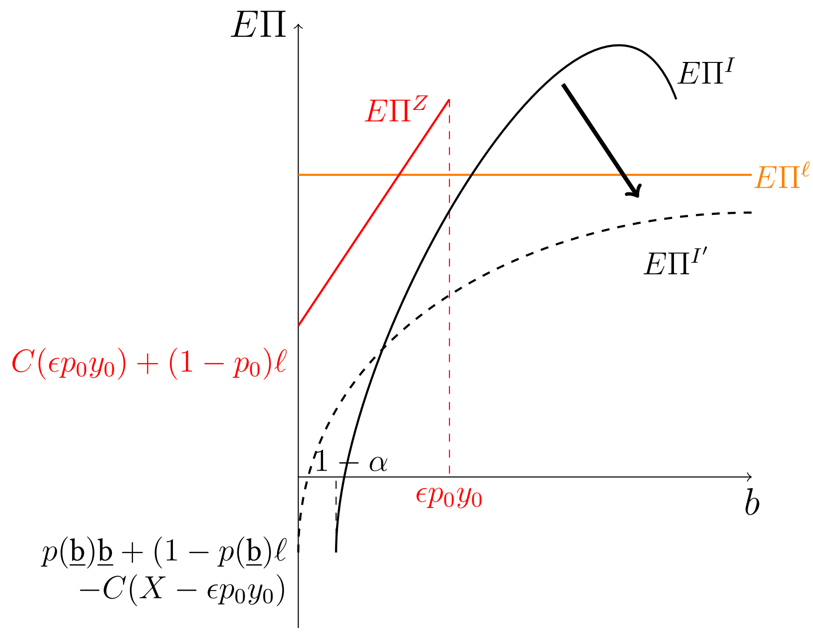

Figure 3: Profit functions for each strategy of the bank. The arrow shows the flattening effect on $E\Pi^I$ of increasing α.

Theorem 3.8 states that when there is a high degree of risk shifting the bank may find it more profitable to keep the firm alive without investing even if the firm has a profitable project. This is the case when disruption costs are high (i.e, low ϕ) or when the shock to the production technology of the firm is low (high ϵ).

In Figure 3, expected profits from funding investment are flattened by an increase in α, meaning a higher scope for risk shifting. Increasing α also increases the lower bound of b, $\underline{b} = 1 - \alpha$ such that the probability is well defined. The amount that can be extracted by zombie lending is unchanged. Note that increasing α also changes both functions via a change in ℓ, but is not depicted. The bank would like to extend funds in normal situations, given that there is a profitable project. However, if the bank extends the funds, the firm would be increase risk to a point in which is not profitable anymore. Without funding investment, the bank can take all of the output. If disruption costs are high, the bank is not willing to liquidate the firm. In terms of the figure, increased disruption costs shift the liquidation profit function downwards.

Notice that this fits into the empirical counterpart used by the literature for zombie-lending, even if revenue is positive. The reason is that, when taking into account the roll over costs the firm's profits are indeed negative. The original debt, D_0, is a sunk cost at this point but made the firm insolvent. Thus, given that $D_0' \equiv \xi \geq r^Z = \epsilon p_0 y_0$ as per Proposition 3.2, then the firm is indeed receiving subsidized lending.

3.5 The Case of a Financially Distressed Bank

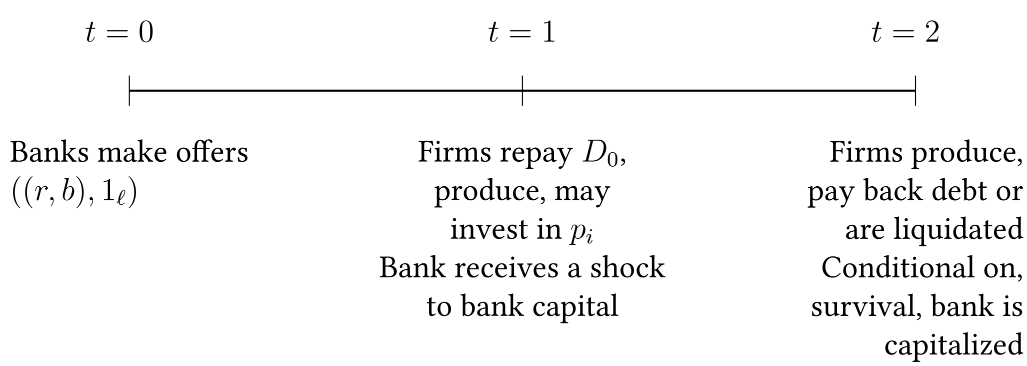

Figure 4: Timing of the model with a financially distressed bank.

The empirical evidence points to the fact that undercapitalized banks are more likely to keep the firms in their portfolios as zombies. To explore this possibility, consider a simple modification to the game, as in Figure 4. Assume now that the bank has capital of A in its books and there are minimum capital requirements, K. If book capital is lower than capital requirements, $A < K$, the bank goes bankrupt and obtains a value of zero. Moreover, assume that there is a shock, ψ to bank capital in period 1 such that $A' = A - \psi$. The shock ψ is distributed according to a smooth accumulated distribution with support on the positive domain, $G(\cdot) \in [0, \infty]$. This can push the bank beyond the minimum capital requirements and thus causes bankruptcy risk. If the bank liquidates the firm in period 0, it would acknowledge the loss of assets in its books. I assume that the bank will be capitalized at the end of the game if it decides to keep the firm afloat instead of liquidating in period 0.[5] Therefore, bank's profits if it decides to liquidate in period 1 are

$$E\Pi^{\ell} = C(\ell)\left(1 - G\left(A - K\right)\right)$$

Where A is declared book capital. If the bank does not recognize its loss, assets will be the original A_0. By liquidating the firm, the bank writes down the value of the assets to $A_0' \leq A_0$. In this case, if the bank liquidates the firm in period 0, it gets instantly the liquidation value, ℓ. This will earn $C(\ell)$ in period 2, since it can be allocated to the alternative investment. However, expected profits are now weighed by the probability of bank survival; i.e, the probability that the shock is smaller than the difference between declared assets and minimum capital requirements.

The following proposition states that when banks are undercapitalized, the conditions for zombie lending are relaxed. In particular, the restriction in Theorem 3.8, on disruption costs and investment costs is no longer relevant. This is because the value of liquidation is less appealing for the bank when undercapitalized, as it increases its own bankruptcy risk.

Theorem 3.9. *(Zombie Firms with undercapitalized banks). Under debt overhang as in Proposition 3.2, when $\alpha \to 1$, then, there exists a level of assets $\bar{A}$ such that, for all $A' \leq \bar{A}$ the bank maximizes profits by offering (r^Z, b^Z) for any $(\epsilon, \phi, RX) \subset [0,1]^3 \times I_X$.*

Figure 5, shows the effect of acknowledging losses, shifting downwards the expected revenue function from liquidating. The schedule of funding investment is unaffected, as well as the schedule for zombie-lending.

4 Policy Implications

Now I discuss several policies implications that arise from the model. The following proposition highlights the fact that debt haircuts can be welfare improving.

[5]There are two reasons for this assumption. First, it simplifies the algebra without loosing generalization. Second, in practice, banks normally smooth out losses using provisions. I abstract from the possibility of a firm taking debt speculating with bank bankruptcy.

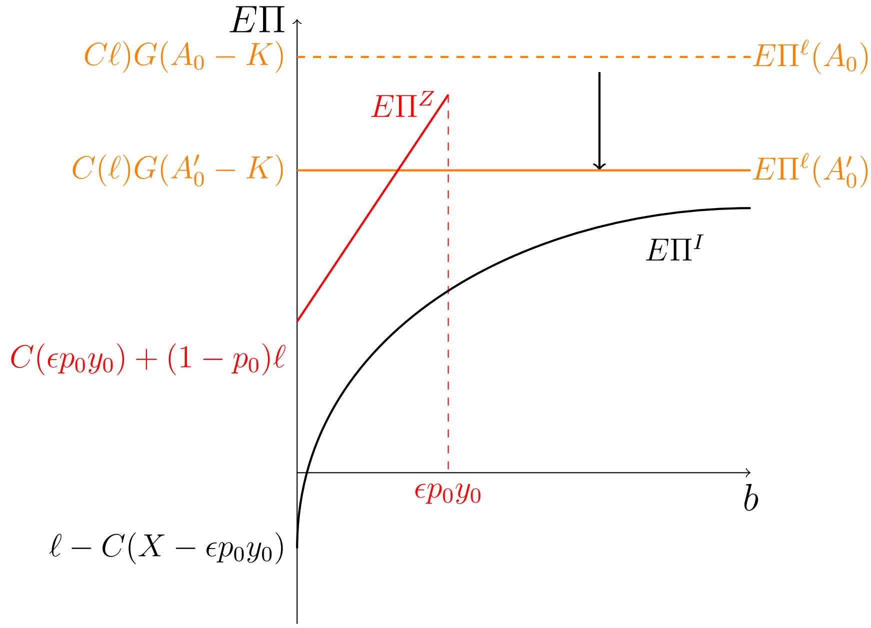

Figure 5: Profit functions of a financially distressed bank when $\alpha \to 1$. The arrow shows the effects of lowering bank's book assets.

Proposition 4.1. *In the presence of zombies as in Theorem 3.8 or Theorem 3.9, there exists a debt haircut $\zeta \in [0, 1]$ on original debt D_0, and lump-sum transfers τ such that at the new level of debt $D_0' = (1 - \zeta)D_0$, the banks and the firm are better off.*

The underlying problem is the presence of moral hazard by the firms, that limit the willingness of the bank to extend new funds. If the bank were to grant funding, the amount that it would have to require on the next period to make it profitable is so large that it is not compensated by the increase in risk. The bank, in turn, prefers to extract all the available funds from the firm for itself. If capitalized, it does not choose to liquidate to avoid the disruption costs. A debt haircut limits the capacity of the bank to extract funds from the firm and aligns the incentives for efficient risk taking. Provided the haircut is large enough, the firms can even regain access to the competitive market. Risk is reduced and expected social surplus increases. This allows the possibility of compensating banks making everyone better off.

Secondly, I analyze monetary policy. A lower interest rate decreases the opportunity cost of funds, thus making banks more prone to lend for investment. This also makes liquidation less desirable, thus partially neutralizing the effect. The next proposition states that there is a set of parameters such that reducing the interest rate has no effect on investment as the as the fundamental incentive problem of risk shifting under debt overhang is too strong. For capitalized banks, this happens when disruption costs are particularly high (low ϕ) or when the shock to the firm is small (high ϵ). The cost of investment, X, is obviously a force against funding investment. The conditions are less stringent for the case of undercapitalized banks, as liquidation is never desirable at sufficiently low asset levels. The next proposition summarizes these results.

Proposition 4.2.

1. *In the presence of zombies as in Theorem 3.9, lowering the interest rate is ineffective*
2. *In the presence of zombies as in Theorem 3.8, lowering the interest rate is ineffective if*
$$\frac{\epsilon}{1+\epsilon} \geq \phi \frac{1}{1+X}(2-X) \tag{9}$$
3. *In the presence of zombies as in Theorem 3.8, if Equation 9 is not satisfied, lowering the interest rate induces higher liquidation if*
$$\phi(1+\epsilon) \geq \frac{1+X}{1-X}$$
for $X \leq 1$.

The last point in Proposition 4.2 states that a decrease in the interest rate may induce higher (inefficient) liquidation but not higher investment if the liquidation value of the firm (which converges to $\phi(1+\epsilon)$) is sufficiently high with respect to the investment cost. I focus on the case of $X \leq 1$, as the firm's revenue is $\lim_{\alpha\to 1} \epsilon p_0 y_0 = \epsilon \leq 1$. In this way, $X \leq 1$ still requires some funds to be extended to the firm. In this case, even if lowering the interest rate may overturn the condition in Theorem 3.8, it will not be enough to resume investment or induce the efficient outcome.

Lastly, I discuss the role of bank recapitalization. Bank recapitalization is not enough to decrease the number of zombies under Theorem 3.8, since bank capital has no role. It could potentially have an effect under the conditions in Theorem

3.9. However, even though recapitalization does affect the liquidation decision, it does not fix the trade-off between zombie-lending and funding investment. Note that liquidation is socially inefficient, since firms have a profitable project and the disruption costs lower the social surplus. When there is zombie lending, there is a hierarchy in banks' strategies, in which the bank first prefers to zombie-lend to other strategies. Bank recapitalization does not increase the incentive to fund investment given that risk shifting and debt overhang are still present. Therefore, when there are zombies, bank recapitalization is either ineffective or induces socially inefficient liquidation.

Proposition 4.3. *In the presence of zombie firms as in Theorem 3.8, bank recapitalization is ineffective. In the presence of zombie firms as in Theorem 3.9, a small bank recapitalization is ineffective, and a large recapitalization may induce inefficient liquidation.*

5 Discussion and Conclusion

Zombie-lending is a phenomenon that has been observed in several countries after the burst of a bubble. This paper explains why zombie-lending arises and continues to thrive, even when renegotiation is possible and efficient projects are available. The model captures a fundamental conflict between lender and borrower in a double-decked incentive problem. The main drivers of this result are that the borrower is locked-in a lending relationship with its incumbent bank and that it has access to a risk-shifting technology. The incumbent bank can extract more funds than socially optimal from the firm. Firms are not be willing to use the fresh funds efficiently, and the bank anticipates this. Notice that firms in the model become suddenly too indebted, but are still viable at lower debt levels. The model is consistent with several stylized facts presented in Section 2.

The model has non-trivial policy implications in line with empirical findings. Debt haircuts are shown to be necessary to restore investment. Fukuda and Nakamura [2011] show empirically that debt relief and capital reduction were important for the recovery in Japan. The model also explains why bank capitalization is not enough to restart investment. Strengthening banks is insufficient if insolvency regimes are hostile to the reorganization of indebted firms, as it only attacks one side of the incentive problem. This is consistent with empirical findings for Europe [Acharya et al., 2016] [Andrews and Petroulakis, 2019]. Finally, the model explains the ineffectiveness of monetary policy to reignite investment without the need of the zero lower bound. The model can be extended to infinite periods allowing for firm entry, in a similar vein as Caballero et al. [2008]. In this case, if the bank does not take into account the effect of zombie-lending on increasing the value of resources, it may deter potential entrants. This includes an externality on the social costs, and thus policies would have a larger impact on welfare.

Several assumptions are essential to obtain the results. First, in the model, debt contracts are non-contingent. Investment is non-contractible and therefore the contract must take into account the scope for risk shifting. The result would not hold if the bank could make contracts conditional on specific projects. However, as long as the bank's monitoring technology is not perfect, the results should hold to a certain degree.

Second, bank opacity is necessary for the case of undercapitalized banks. Otherwise, investors would price the asset loss in the market value of the bank.[6] It could be argued that regulators should have superior information and thus could put sanctions direclty. Hellwig et al. [2012] and Hoshi and Kashyap [2004] discuss the issue of regulatory forbearance, which is assumed away in this paper. The main results hold even even with no regulatory forbearance and capitalized banks, provided disruption costs are substantial.

Third, I assume that the firm has debt with only one bank.[7] I abstract from potential conflicts of interest between lenders to highlight the debt overhang channel. Notwithstanding, syndicated loans have been growing increasingly more common, which may cause a large number of creditors to behave collusively.[8]

Lastly, I assume that voluntary bankruptcy is not allowed or, equivalently, that is not preferred to remaining a zombie. Otherwise, an arrangement as Chapter 11 should restore efficiency.[9] In a way, my model assumes market incompleteness for renegotiation and/or a stigma associated with bankruptcy.[10] Large cross-country differences in these aspects may explain the relative prevalence of

[6] BIS [2011], Flannery et al. [2004] and Huizinga and Laeven [2012] provide evidence of opaqueness.

[7] Noe and Wang [2000] and Bolton et al. [1993] focus on the role of many creditors.

[8] See Gadanecz [2004] and Esty and Megginson [2003].

[9] See for example Annabi et al. [2012] and Franks and Torous [1994].

[10] Evidence of this can be found in Semadeni et al. [2008] and Gilson and Vetsuypens [1993].

zombies found in papers such as Andrews and Petroulakis [2019].

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6 Proofs

Proof of. Proposition 3.2. First we show that If D_0 is lower than $\bar{D}$, both banks offer $r = -X$ and $\hat{b} = RX + \frac{1-p'}{p((\hat{b})}\ell' - \frac{1-p(\hat{b})}{p(\hat{b})}\ell$. Assume it is not an equilibrium offer. If $r \neq -X$, there is either not sufficient funds for investing or idle funds, so it is not an equilibrium. If the incumbent bank offers $\hat{b}' < \hat{b}$, then $\Pi < 0$, thus contradicting bank's incentive compatibility. If the incumbent bank offers $\hat{b}' > \hat{b}$, then the competitor can offer $\hat{b}'' = \hat{b}' - \varepsilon$, for arbitrarily small ε and have its offer accepted. Since there are enough funds to invest and the project is profitable, firm reacts using Equation (4) thus doing the project $p(\hat{b})$. Lastly, if the incumbent bank offers to liquidate, $\mathbb{1}_\ell = 1$, then the firm gets zero profits. Then, the competitor can make an offer $(-X, \hat{b} - \varepsilon)$ such that $\Pi > 0$ and $\pi > 0$. Same logic applies to the competitor making offers. Thus, it is not an equilibrium.

If $D_0 > \bar{D} \equiv \kappa\{(y_0 - r - X(I) + \max_i\{EV_i(b), y_0\}\} \geq y_0$, where $\kappa > 1$. The claim is the bank will offer $(r^I, b^I, \mathbb{1}_\ell^I)$ maximizing profits as in Equation 1 subject to Equation 4 and Equation 7. Suppose it is not an equilibrium. Then the incumbent bank can increase profits by offering $b^I + \epsilon$, which contradicts the maximization. The competitor bank can never make an offer below b^I, since Equation 6 never holds. Since $\pi(b) > 0$ for all b, then the firm is always strictly better accepting any offer from the incumbent bank. $\square$

Proof of. Proposition 3.5. If Assumption 3.3 holds, then $\hat{b} = RX$. By replacing in Equation 4, we obtain the desired result. The indirect profit function for the firm is given by

$$E\pi(b) = \left(\frac{1-\alpha}{b}\right)^{\frac{1-\alpha}{\alpha}} - \left(\frac{1-\alpha}{b}\right)^{1/\alpha} b$$

Outside value of not investing is $p_0 y_0$. By replacing $\hat{b} = RX$ and operating, we find that if $RX \in I_X$ investment is preferred. Probability is well defined under Assumption 3.4 since $RX \geq 1 - \alpha$. Maximizing social surplus is equivalent to maximizing welfare,

$$\max_p W(p) = p^{1-\alpha} - C(X) + (1-p)\ell = p^{1-\alpha} - RX\hat{p}$$

which under Assumption 3.3 is maximized at the competitive level, $\hat{p}$. $\square$

Proof of. Proposition 3.6. First I show that each of these strategies is an equilibrium. Then, I show that these are the only possible strategies.

1. First we show that (r^I, b^I) is an equilibrium strategy if the bank wants to fund investment. $r^I = \epsilon y_0 - X$ is an equilibrium given that any $r' \neq r^I$ is either not enough to fund investment or leaves idle resources. To find b^I, replace the reaction function of the firm in the problem of the bank and maximize,

   $$\Pi = \left(\left(\frac{1-\alpha}{b}\right)^{1/\alpha} b + (1 - \left(\frac{1-\alpha}{b}\right)^{1/\alpha})\ell + C(r)\right) = (1-\alpha)^{(1-\alpha)} b^{\frac{-1}{\alpha}}(b-\ell)$$

   The maximization yields,

   $$-(1-\alpha)^{\frac{1}{\alpha}} \frac{b^{\frac{-1}{\alpha}-1}}{\alpha}(b-\ell) + (1-\alpha)^{\frac{1}{\alpha}} b^{\frac{-1}{\alpha}} = 0$$

   which simplifies to $b^M = \frac{\ell}{1-\alpha}$.

2. If the bank decides to zombie-lend, it sets (r^Z, b^Z). Assume, on the contrary, that $r > \epsilon p_0 y_0$, then the firm makes negative profits, thus violating feasibility. Assume $r < \epsilon p_0 y_0$. Since bank's profits are increasing in r, $r' = r + \varepsilon$ with arbitrary small ε, is feasible and yields higher profits.

3. Liquidating is trivially a strategy the bank finds it profitable to liquidate.

Say the bank offers r'' such that $r^Z > r'' > r^I$, then there are either not enough funds to invest (in which case it cannot extract b^I) or idle funds. Thus, only possible equilibrium strategies are $\{(r^Z, b^Z), (r^I, b^I), \mathbb{1}_\ell\}$. □

Proof of. Lemma 3.7. From Proposition 3.5, all we need to show now is that $\hat{p} > p^I(b^I)$. Since $(1-\alpha)^2 \geq \ell$, and $RX \geq (1-\alpha)$, this is true. Under Proposition 3.4, the project at $\hat{p}$ is efficient and maximizes welfare at $W(\hat{p})$. Thus, welfare is lower if the bank funds investment at $W(p^I(b^I))$. If the bank zombie-lends then $W(p_0) \leq W(\hat{p})$. Liquidation is inefficient since implies disruption costs and missed efficient projects. □

Proof of. Theorem 3.8 First, we start with conditions under which $E\Pi^Z \geq E\Pi^I$ (Part A). Then, we find conditions under which $E\Pi^Z \geq E\Pi^\ell$ (Part B).

Part A. Bank will prefer to offer $r = \epsilon p_0 y_0$ and $b = \epsilon p_0 y_0$ instead of funding investment if

$$C(\epsilon p_0 y_0) + \epsilon p_0 y_0 + (1-p_0)\ell \geq C(\epsilon p_0 y_0 - X) + p(b)b + (1-p(b))\ell$$

Replacing and simplifying,

$$0 \leq \hat{p}RX + \epsilon p_0^{1-\alpha} - p_0 \ell - \alpha\left(1-\alpha\right)^{\frac{2-\alpha}{\alpha}} \ell^{-\frac{1-\alpha}{\alpha}} \equiv f\left(p_0, \alpha, \phi, \epsilon, RX\right) \tag{10}$$

Where we introduce $f\left(\cdot\right)$. Our aim is to characterize the domain under which it is positive valued. To make sure that probabilities are well defined, $\ell \geq (1-\alpha)^2$ and $RX \in I_X$, the latter of which also ensures efficiency. Therefore, we need to characterize the parameters for which $f \geq 0$ under following conditions

$$\begin{aligned}
& y_0 = \epsilon p_0^{-\alpha} \in \left(0, \alpha\right], \\
& \ell = \phi\left(p_0 y_0 + \alpha\left(1-\alpha\right)^{\frac{1-\alpha}{\alpha}} \left(RX\right)^{-\frac{1-\alpha}{\alpha}}\right) \geq \left(1-\alpha\right)^2 \\
& RX \in I_X \equiv \left[(1-\alpha), (1-\alpha)\left(\frac{\alpha p_0^{\alpha}}{\epsilon}\right)^{\frac{\alpha}{1-\alpha}}\right] \\
& p_0, \epsilon, \phi, \alpha \in \left[0, 1\right]
\end{aligned}$$

Thus, (10) holds in the domain $\Omega = \left\{(\alpha, \phi, \epsilon, RX) \in \left[0,1\right]^3 \times I_X, \right\} \subset \mathbb{R}^5$.

Claim # 1:

$$\lim_{\alpha \to 1} \hat{p} \to \frac{1}{RX}$$

Proof. Let $X \in I_X = (1-\alpha) + x$. Then, $RX = R(1-\alpha) + Rx \in I_X$. Then,

$$\lim_{\alpha \to 1} \hat{p} = \lim_{\alpha \to 1} \left(\frac{1-\alpha}{R(1-\alpha) + Rx}\right)^{1/\alpha} = \frac{1}{Rx} = \frac{1}{RX}$$

Claim # 2:

$$\lim_{\alpha \to 1} I_X \to \left[0, +\infty\right]$$

Proof. Taking limit on the bounds of I_X, $\lim_{\alpha \to 1}\left(1-\alpha\right)\left(\frac{\alpha p_0^{\alpha}}{\epsilon}\right)^{\frac{\alpha}{1-\alpha}} = +\infty$

Claim # 3:

$$\lim_{\alpha \to 1} \ell = \phi\left(1+\epsilon\right) \tag{11}$$

Proof. Simplifying ℓ,

$$\ell = \phi\left(p_0 y_0 + \alpha\left(1-\alpha\right)^{\frac{1-\alpha}{\alpha}} \left(RX\right)^{-\frac{1-\alpha}{\alpha}}\right) = \phi\left(\epsilon p_0^{1-\alpha} + \alpha\left(\frac{1-\alpha}{RX}\right)^{\frac{1-\alpha}{\alpha}}\right)$$

Since $\lim_{\alpha\to 1}\alpha\left(\frac{1-\alpha}{RX}\right)^{\frac{1-\alpha}{\alpha}}=1$, and $\lim_{\alpha\to 1}p_0^{1-\alpha}=1$, result holds.

Now we find a subset $\Omega'\in\Omega : f\geq 0$. Taking limits term by term and using results from Claims #1 and #3,

$$\begin{aligned}
&\lim_{\alpha\to 1}\hat{p}RX=1\\
&\lim_{\alpha\to 1}\epsilon p_0^{1-\alpha}=\epsilon\\
&\lim_{\alpha\to 1}p_0\ell=p_0\phi(1+\epsilon)\\
&\lim_{\alpha\to 1}\alpha\left(1-\alpha\right)^{\frac{2-\alpha}{\alpha}}\ell^{-\frac{1-\alpha}{\alpha}}=0
\end{aligned}$$

Since $p_0,\phi\leq 1$, then $1+\epsilon\geq p_0\phi(1+\epsilon)$, and $\lim_{\alpha\to 1}f\left(\alpha,\phi,\epsilon,RX\right)\geq 0$ for any $\Omega'\in\Omega$.

Part B. If $E\Pi^Z\geq E\Pi^\ell$,

$$C(\epsilon p_0y_0)+\epsilon p_0y_0+(1-p_0)\ell\geq C(\ell)$$

Replacing and taking limits, when $\alpha\to 1$ Operating,

$$\frac{\epsilon}{1+\epsilon}\geq\phi\frac{1}{1+X}\left(1-X+\frac{1}{R}\right)$$

□

Proof of. Theorem 3.9. $E\Pi^Z$ and $E\Pi^I$ are not affected by the modified version of the game. Therefore, from Theorem 3.8, bank prefers to offer (r^Z,b^Z) to (r^I,b^I) when $\alpha\to 1$. Now we need to find conditions such that $E\Pi^Z\geq E\Pi^\ell$. Since $G\left(A-K\right)$ is continuously non-increasing in A, then, Π^L is continuously non-decreasing in A'. When $A\longrightarrow K$, $G\to 0$ and $E\Pi^\ell\to 0\leq E\Pi^Z$. □

Proof of. Proposition 4.1. Welfare is maximized under competitive markets from Proposition 3.5. From Proposition 3.2, market structure depends on the level of debt. Suppose debt is $D_0=\xi$. A haircut $\zeta>0$ turns debt into $D_0'=(1-\zeta)\xi<\bar{D}(y_0)$. From Proposition 3.7, welfare is non decreasing. Increased surplus can be redistributed in a lump sum fashion using transferences τ such that both agents are better off. □

Proof of. Proposition 4.2. .

1. Theorem 3.9 holds for any $RX \in I_X$, therefore changing R does not change the inequalities.

2. By setting $R = 1$ in Equation (8), we obtain the result.

3. If (8) is not satisfied but Theorem 3.9 is, it must be that

$$\phi \frac{1}{1+X}(1-X) \leq \frac{\epsilon}{1+\epsilon} \leq \phi \frac{1}{1+X}(2-X) \tag{12}$$

   For liquidation to be preferable to investment we need to show that $\lim_{\alpha \to 1} E\Pi^{\ell}(R = 1) \geq \lim_{\alpha \to 1} E\Pi^{I}(R = 1)$. By taking limits in profit functions, $\lim_{\alpha \to 1} E\Pi^{I} = \frac{\epsilon}{X} + \phi(1+\epsilon)\left(2 - \frac{1}{RX}\right) - 1$, and $\lim_{\alpha \to 1} E\Pi^{\ell} = \phi(1+\epsilon)\left(\frac{1}{X} + 1 - \frac{1}{RX}\right)$. Therefore, $\frac{\partial E\Pi^{I}}{\partial R} = -\frac{\phi(1+\epsilon)}{R^2 X} = \frac{\partial E\Pi^{\ell}}{\partial R}$. Thus, if

$$\epsilon + \phi(1+\epsilon)(X-1) \leq X \tag{13}$$

   then liquidation is preferred. All we need to find conditions under which the set in which Equations 13 and 12 is non empty. The set of inequalities simplifies to proving that the following holds

$$\phi \frac{1-X}{1+X} \leq \frac{X}{1+\epsilon} - \phi(X-1)$$

   Operating, we find that this is the true when $\phi(1+\epsilon) \geq \frac{1+X}{1-X}$ if $X \leq 1$ and $\phi(1+\epsilon) \leq \frac{1+X}{1-X}$ if $X \geq 1$, both of which are non empty for $\phi, \epsilon \in \Omega$.

□

Proof of. Proposition 4.3. Under Theorem 3.8, the level of assets of banks is irrelevant, so it is trivially true. Under Theorem 3.9, There are two cases. First, if $E\Pi^{Z} \geq C(\ell)$, the bank prefers to zombie lend at any level of bank assets since $(1 - G(A - K)) \leq 1$, and thus capitalizations are not effective. Second, $C(\ell) \geq E\Pi^{Z} \geq C(\ell)(1 - G(A_0 - K))$ where A_0 are current bank assets. Given the continuity and monotonicity of G, there is a level of bank assets $\bar{\bar{A}} \geq A_0$ such that $C(\ell)G(\bar{\bar{A}} - K) = E\Pi^{Z}$. Thus, for for all recapitalizations ΔA such that $A_0 + \Delta A \geq \bar{\bar{A}}$, bank prefers to liquidate rather than zombie- lend. Liquidation is inefficient from Proposition 3.5. □