Banks vs Zombies

Nicolas Aragon [†]

Abstract

After bubbles collapsed, banks have often rolled-over debt at subsidized rates to insolvent borrowers or "zombie firms." I study the incentives to restructure debt in a game with risk shifting under debt overhang. I provide conditions under which it is optimal to zombie-lend even when socially inefficient. When a creditor becomes insolvent, the firm loses access to competitive funding and instead, its incumbent bank can exert monopoly power. The bank prefers to zombie-lend given that liquidating the firm destroys value, and funding investment requires a large repayment which decreases overall expected repayment due to risk-taking. I also study the case of a financially distressed bank and discuss monetary policy, bank recapitalization and debt haircuts.

JEL Classification Codes: G2, G21, G28, G33, G32

[†]Universidad Carlos III de Madrid, National Bank of Ukraine and Kyiv School of Economics. Email: naragon@kse.org.ua. I would like to thank David K. Levine and Árpád Ábrahám for comments, insights and guidance. I am also indebted to Marco Celentani, Andres Erosa, Marce Fernandez, Pablo Forero, Willy Fuchs, Martin Guzman, Daniel Heymann, Hugo Hopenhayn, Anton Korinek, Ramon Marimon, Rasmus Pank, Guillermo Ordonez, Facundo Piguillem, Francisco Roldan, Luis Rojas, and Emanuele Tarantino for comments that greatly improved this paper.

1 Introduction

Movements in asset prices can destroy collateral values and spell trouble for both banks and firms, to the point of insolvency. Insolvent firms may continue to operate if banks roll over their debt at subsidized rates. These "zombie firms" survive but struggle to grow or invest in new projects. This phenomenon arose in Japan in the early 90s and led to widespread misallocation of resources, and a lower productivity of the economy [Caballero et al., 2008]. Debt renegotiation should, in principle, lead to both parties reaching a welfare improving situation in the face of debt overhang. However, the experience shows that this is not necessarily the case.

Zombie-lending is not exclusive to Japan. The phenomenon has also been observed in Europe in the last years. Several features are common to these crises. In particular, the number of zombie firms spikes after the burst of an asset bubble, and they are often in a lending relationship with a poorly capitalized bank. Moreover, these firms are less likely to invest. From a policy side, bank recapitalization and lowering the interest rate proved unsuccessful in igniting growth (see Section 2).

I analyze the incentives of lenders and borrowers to explain why zombie-lending continues to thrive and is not solved by the market. Throughout the paper zombie-lending is defined as the practice by a bank to continue a lending relationship with an insolvent firm instead of liquidating it. A firm receiving this type of lending will be considered a zombie-firm, and will endogenously cease to invest, even when profitable projects are available.

I consider a bank-firm game with two main ingredients: risk shifting and debt overhang. In this setup, I find a set of conditions under which zombie-lending arises as an equilibrium outcome, even when debt restructuring is allowed. Due to debt overhang, firms are financially distressed and locked in with their incumbent bank. I separate the cases cases of positive operational revenues and negative operational revenues (i.e, without taking into account interest payments); and the cases of a capitalized bank and a non capitalized bank. I extract policy implications regarding monetary policy, debt haircuts and capital injections for each case (Section 4). Importantly, the effect of the interest rate depends on the nature of the firms receiving subsidized lending. The predictions of the model, as well as the policy implications are aligned with stylized facts (Section 2).

The main mechanism of the model is that a firm that suddenly becomes insolvent looses access to the competitive credit market, becoming locked in with its incumbent bank. This creates a disconnection between the incentives of the bank and the firm. The bank can thus extract a larger surplus from the captive firm. On the other hand, if a firm would be granted a loan to enact an efficient project by the bank, the repayment for profit maximization would need to sizeable. The firm would then, overburdened, not to use the funds efficiently and instead increase the risk in their investment project. Anticipating the firm's behavior, the incumbent bank will not extend fresh funds. Moreover, the bank will decide not to liquidate the firm because the disruption costs are large, or because such a move would force it to acknowledge losses in its balance sheet and increase the risk of becoming bankrupt itself.

The model consists of three agents: a firm and two banks that play for three periods. The firm starts with an exogenous level of debt contracted with one bank, the incumbent. The firm has access to a continuum of projects differing in risk. In normal times, when initial debt is low, the firm can repay its incumbent bank, and borrow from any of the two banks to finance investment. Bank competition pushes the cost of borrowing until banks make zero profits. The firm invests in a project with socially optimal risk and social welfare is maximized.

In crisis times, the firm becomes insolvent and banks may become distressed by a shock. I model the crisis as a shock of zero measure and assume that firms will never become solvent at any point in the future. If it were not the case, firms could attract funds from other investors. Following from the assumption that old debt is senior to new debt, a hold-up problem arises. The firm is no longer able to find funds in the competitive market, and the incumbent bank becomes a monopolist.

As a monopolist, the bank has two broad strategies in hand: it can liquidate the firm and put the money in a risk free technology, or extend credit. The firm, in turn, can use the funds at its own discretion. At the end of the game, the bank is either repaid or it liquidates the firm. In this model, I show that there are various situations under which the bank decides to keep the firm in the market, even if financially or operationally insolvent. In other words, the bank zombie-lends. Importantly, this can happen when the firm has a profitable project and would be socially beneficial to restructure debt.

The bank will decide not to extend enough funds for the investment project when there is large scope for risk shifting. This is because any funds extended to a distressed firm will end up being "gambled" with low expected returns for the bank. If the bank liquidates the firm, it must pay disruption costs. In response, the bank prefers to take the business-as-usual revenue stream for itself.

I consider the cases of a financially distressed firm and with positive and negative operationally revenue. A financially distressed even though has positive operational revenues and net present value, is unable to meet the interest payments. A financially distressed firm with negative operational revenues (i.e, operationally distressed). I will focus on the cases when operationally distressed firms may still have available efficient investment projects and positive net present value. I show that the conditions for zombie-lending and the effects of policy in each case are substantially different.

I also analyze the case of a financially distressed bank and firm simultaneously. I model this in a setting where a bank may suffer from a shock to its assets. If the bank decides early liquidation of the firm, it will face a shock in period 1 that may render itself bankrupt. Liquidating a firm implies recognizing lost assets. A bank in a strong capital condition would liquidate the firm and acknowledge the loss in its books. However, a weak bank may not be willing to do so, since it may increase its own bankruptcy risk. Importantly, firms with operational losses will become zombies only if banks are financially distressed.

The model yields several policy prescriptions. First, a haircut on firm debt can increase welfare. A firm in debt overhang is overburdened by debt, and thus decides to increase the risk of its project. A haircut on debt decreases the bank's market power, allowing the firm to obtain funds and invest. Social surplus becomes larger and can be used for a redistribution such that the bank is also better off. Second, recapitalization may be either ineffective or induce the inefficient liquidation of the firm. It will not, however, increase investment. Recapitalizing a bank without a haircut on corporate debt does not change the firm's incentives and is not enough to resume investment. The optimal policy response to zombie-lending includes both bank recapitalization and a debt haircut. Third, when risk shifting is prevalent monetary policy can be ineffective. Decreasing the interest rate decreases the cost of investment, but may not be enough to solve the underlying incentive problems.

Related Literature. Several papers dealt empirically with the presence of zombie-lending in Japan since the early 90s, starting with Hoshi [2006]. Caballero et al. [2008] complemented this literature with a theoretical model of the effects of zombie firms on the rest of the economy. My paper relates to this literature by explaining how this phenomenon can arise in a decentralized equilibrium. As such, it is linked to the literature on Debt Forbearance. Hellwig et al. [2012] distinguish different types of forbearance: among the supervisor and the bank, and among the bank and the debtor and Bruche and Llobet [2013] focuses on a bank that can hide a bad loan from the regulators and study the regulatory scheme. I abstract from the incentives of the regulators to solely focus on profit-seeking agents. Berglof and Roland [1998] was the first paper to study troubled firms that fail to exit the market. However, their focus is on state owned companies, and a government that wants to avoid social costs. I, instead, show how competitive markets can lead to the same situation. Overall, a key contribution of my paper is to provide a framework to understand the exact conditions for zombie lending, and the effects of policy under this conditions.

To build this model, I combine insights from three bodies of literature: debt overhang, risk shifting, and renegotiation.

The debt overhang literature highlights that large amounts of debt lead to underinvestment by firms given that the profits would benefit existing debt holders first, instead of the new investors [Myers, 1977]. This work spanned a large literature.[1] My paper relies on this argument and focuses on the incentives of the lenders, and not just the borrowers after a large aggregate shock. Kovrijnykh and Szentes [2007] study sovereign countries in debt overhang focusing on lenders as well. Their setup is very closely related to my paper. My model can be considered a simplified version of Kovrijnykh and Szentes [2007] that focuses on firms. Unlike countries, firms can be liquidated and have large scope for risk shifting. These two differences substantially change the space of strategies and outcomes. Chen and Manso [2017] focuses on debt overhang and investment decisions from a more macroeconomic perspective.

Risk shifting, or the incentives of financially distressed firms to gamble with their assets, was first introduced in Jensen and Meckling [1976]. The evidence is mixed,[2] and the literature has explored theoretically several mitigating factors.[3] My proposed mechanism is also able to account for the elusiveness of the mechanism in the data.

The literature on renegotiation [Hart and Moore, 1998, Hellwig, 1977, Hart and Tirole, 1988] highlights that when there is a bargaining surplus, default is inefficient. As such, my theory relates to delays in this process. Admati and Perry [1987] and Vilanova [2004] explain these delays based on information frictions and relative bargaining powers between borrowers and lenders. In my model, shocks are permanent and the evolution of the firm's output is known by both parties. Moreover, I focus on a case where the bank acquires all the bargaining power, arising from debt overhang. There are several papers focusing on these delays for the case of countries, [Bai and Zhang, 2012, Pitchford and Wright, 2012], but since my paper regards to a firm that can be liquidated, the mechanism changes substantially.

[1] For example, Hennessy [2004], Titman and Tsyplakov [2007], Moyen [2007], Diamond and Rajan [2009], and Occhino and Pescatori [2015].

[2] See for example, De Jong and Van Dijk [2007], Eisdorfer [2008], and Gilje [2016].

[3] See for example, Almeida et al. [2011] and Barnea et al. [1980].

Kahl [2002] addresses a similar question to this paper regarding the inefficiency of the restructuring process. The focus of that paper is on the lack of information form part of the lenders to decide quickly an efficient liquidation. The paper builds a dynamic learning model for creditors. Rajan [1994] provides a model with short-term concerns from a banker with reputational concerns which features a "gambling for resurrection" type of behavior. In my paper, in contrast, there is no learning and the focus is on the mix of incentives of both creditors and lenders.

Several other papers combine insights from one or more of these bodies. Favara et al. [2017] mix the underinvestment result from Myers [1977] with renegotiation. They find that the prospect of easier renegotiations mitigates the underinvestment result. I, on the contrary, focus on banks' opportunistic behavior. Vilanova [2004] develop a model of restructuring for firms in distress, where banks have an option value of waiting for changes in firms' fundamentals. They focus on the banks' opportunistic behavior but, unlike my paper, they do not account for moral hazard by the firms. Moreover, in my model, there is no uncertainty regarding the firm's expected recovery.

Manso [2008] presents a model of risk shifting, in which debt distorts new projects and analyze the effect of allowing for default. This paper does not consider the incentives of the lender and debt restructuring, which are the focus of my paper.

Frantz and Instefjord [2019] combine debt overhang and debt restructuring. Debt restructuring mitigates underinvestment, and is aimed at managing the borrowing policy in non-distressed states. I include risk shifting in a simpler model, which substantially changes results.

Pawlina [2010] shows that shareholders' option to renegotiate debt exacerbates the debt overhang underinvestment result due to the higher wealth transfer after investment. Efficiency is restored by granting creditors the entire bargaining power and the availability of efficient projects alleviates the problem. My results are in stark contrast with this result.

Lastly, a final salient feature of the Japanese and European crises is that, despite lower interest rates and bank recapitalization, investment did not reignite. There is a rich discussion about the desirability of lowering the interest rate in this context.[4] Moreover, current explanations on why banks hoard cash instead of lending it rely on uncertainty. I offer an alternative explanation for the lack of reaction of investment and for cash hoarding based on risk shifting by financially distressed firms.

The paper is structured as follows. Section 2 summarizes results of several empirical papers in the form of stylized facts that motivate this paper. Section 3 presents the model and Section 4 analyzes policy implications. Section 5 discusses the results and assumptions, and concludes.

2 Stylized Facts

Zombie firms were first identified as such in Japan after the burst of an asset and real state bubble. Japan's growth remained stagnant for decades since. Several papers have documented similar features for Europe during the recent crisis. In this section, I highlight a number of stylized facts common to both cases, that motivate and guide my theory.

1. *Zombie firms become more widespread after the burst of a bubble.* Caballero et al. [2008] identify zombie firms in Japan as those receiving a subsidized rate in their loans. This requires high debt. They found zombies to be prevalent, and that its number substantially increased after the collapse of the bubble (Figure 1). Aragon [2018] finds the same pattern for Spain.

2. *Highly indebted firms have a higher likelihood of being associated with a financially distressed bank.* Peek and Rosengren [2005] find that undercapitalized banks are more likely to lend to poorly performing firms in Japan. Andrews and Petroulakis [2019] present cross-country evidence for European countries of this relationship. This is particularly apparent in countries in

[4]See Krugman et al. [1998], and Obstfeld and Duval [2018], among others.

which insolvency procedures inhibit corporate restructuring.

3. *Highly indebted firms are less likely to invest.* This is a classic debt overhang result with a large empirical literature supporting it. For example, Peek and Rosengren [2005] and Kalemli-Ozcan et al. [2018].

4. *Zombie firms and firms with high debt have negative effects on the economy.* Several channels were explored empirically: slowdown in productivity [Andrews and Petroulakis, 2019], misallocation of resources Gopinath et al. [2017], declining business dynamics [Decker et al., 2018]. Caballero et al. [2008] and Ahearne and Shinada [2005] show that zombie firms had negative effects on job creation, as well as on non-zombie's productivity.

5. *When zombies are widespread, bank recapitalizations had mild effects on activity.* Acharya et al. [2016] analyze the real effects of the Outright Monetary Transactions Program, which increased the price of European sovereign debt, thus recapitalizing banks. They find that banks that were recapitalized indeed increased their lending but directed these funds towards troubled firms, that failed to invest. Schivardi et al. [2017] and Giannetti and Simonov [2013] present similar results. Giannetti and Simonov [2013] shows that in Japan, mild recapitalizations did not increase investment.

6. *When zombies are widespread, investment did not increase despite low interest rates.* Interest rates have remained close to zero for both Europe and Japan, but investment did not resume [Krugman et al., 1998, Eggertsson et al., 2014].

To summarize, these facts present important regularities about zombie firms (receiving subsidized lending) and over-indebted firms after the collapse of a bubble. I include facts regarding firms with large debt because this is a necessary condition for a firm to receive zombie-lending in my theory, and few data sets record interest payments. The first fact motivates the use of a zero measure shock, given that collapses of bubbles must be unexpected. I will next present a theory that is consistent with the remaining regularities, combining risk shifting and debt overhang arguments.

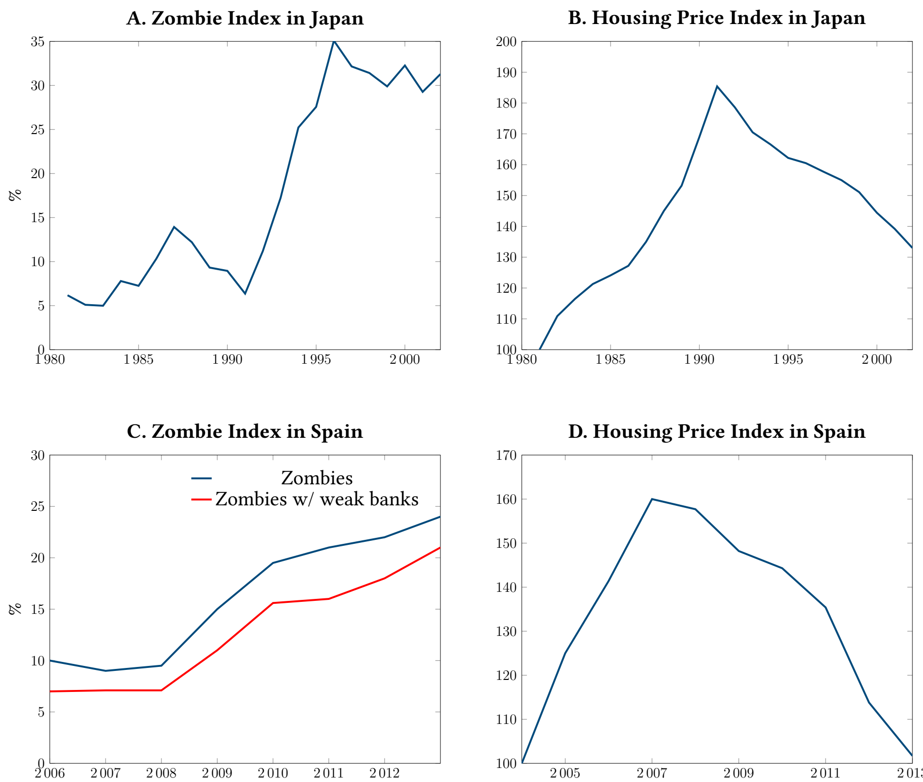

Figure 1: Percentage of Zombie Firms and Housing Price Index in Japan and Spain. Source: Aragon [2018] and Caballero et al. [2008].

3 The Model

I present a simple game that provides theoretical insights on the behavior of borrowers and lenders in a situation of debt overhang when risk shifting is possible. I show that once a a situation of debt overhang arises, the decentralized renegotiation equilibrium may be inefficient. Furthermore, I show that an equilibrium exists where firms fail to invest in available profitable opportunities even though banks could finance them. I also analyze different policies and show how they can implement an efficient allocation.

3.1 Environment

There are three risk neutral agents: a firm and two banks. They live for three periods: 0, 1, 2. The firm seeks resources to finance a risky project in a lending market and has an inherited debt with one of the banks, the incumbent. In period 0, banks compete *á la* Bertrand to provide funds to the firm, and offer a pair of payment in period 1 and payment in period 2. A bank can also offer to liquidate the firm in period 1 (and put the proceeds in a risk-free technology) or in period 2. The firm may choose to accept or reject these offers.

The firm possesses a technology to produce output. In period 1, the firm produces using its traditional technology. If it has enough funds, it may also carry on an investment project. The investment project, if successful, changes the technology of the firm, which is used in period 2 to produce and pay back debt. If the firm fails in period 2, the firm is liquidated and its assets sold. The timing of the game is in Figure 2.

Banks. There are two banks: the incumbent bank and the competitor bank. The incumbent bank has a claim on firm debt D_0. In period 0, each bank can offer two types of contracts: flowing funds or liquidation. The banks can offer to immediately liquidate the firm in period 1 or to liquidate in period 2. We denote all contracts as a pair for the actions in period 1 and 2 respectively (σ_1, σ_2). The banks have a riskless investment opportunity that pays Rx with $R \geq 1$ for any amount of invested funds x.

Each bank can offer a pair (r, b) composed by a payment r in period 1, and a payment of b in period 2. If the firm cannot repay in period 2, it will be liquidated. Note that r can be either greater than zero (in which case the bank charges an amount) or smaller than zero (in which case the bank extends funds).

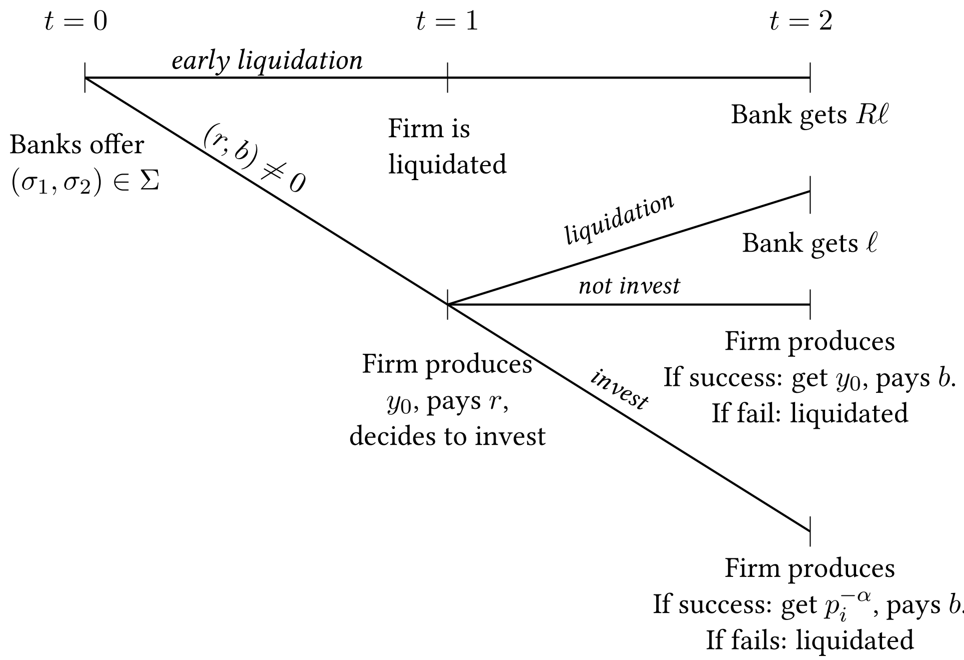

Figure 2: Timing of the model

The banks can offer to liquidate the firm in period 1 and put the proceedings in a risk free technology. In period 2 the bank will receive zero from the firm under this strategy as the firm no longer exists. In this case, the bank's strategy will be $(\mathbb{1}_\ell = 1, 0)$, where $\mathbb{1}_\ell$ is an indicator function equal to one if the bank liquidates in period 0 and zero otherwise. Clearly, no firm that can repay its original debt would choose to accept this offer from its incumbent bank. Moreover, if the competitor bank, with no claims on the firm, would make this offer, it would never be accepted. Therefore, this strategy will only be used by the incumbent bank when the firm is insolvent.

The banks may choose to extend or receive funds from the firm in the first period, r^P, and liquidate in the second.[5] This contract is denoted as $(r^P, \mathbb{1}_\ell = 1)$.

[5] I assume that in case of late liquidation, assets are sold to a different entrepreneur that can use them for two periods.

The incumbent bank's expected profits, $E\Pi$, of offering a contract given by

$$E\Pi(\sigma_1, \sigma_2) = \begin{cases} R\ell & \text{if } (\sigma_1, \sigma_2) = (\mathbb{1}_\ell = 1, 0) \\ r^P + \ell & \text{if } (\sigma_1, \sigma_2) = (r^P, \mathbb{1}_\ell = 1) \\ p(b)b + (1 - p(b))\ell + Rr & \text{if } (\sigma_1, \sigma_2) = (r, b) \neq 0 \end{cases} \tag{1}$$

conditional on the contract being accepted by the firm. In Equation 1, the liquidation value of the firm is $\ell = \max\{\phi E\pi, 0\}$, where $E\pi$ is the net present value of the firm under competitive markets, and $1 - \phi$ are the liquidation disruption costs. Notice that if the firm's profits are negative, the liquidation value is zero. When the bank decides to do an early liquidation of the firm, it will receive $R\ell$, due to earnings in the riskless technology. If the bank decides to do a late liquidation, it will receive ℓ but will also earn r^P from the first period. Finally, if the bank decides to extend new funds, it will get r in the first period that will go into the riskless technology earning Rr, and b in the second period, conditional on the firm being successful. Firm's success in a project is given by $p(b)$. If the project is unsuccessful, the bank will get its liquidation value, ℓ. The bank understands that different repayments b will provide different incentives for the firm, which translate into different success probabilities, $p(b)$.

The competitor bank's potential strategies are denoted by primes: (σ_1', σ_2'). The potential strategies are the same as the incumbent bank: extend loans for both periods, (r', b'), immediate liquidation, $(\mathbb{1}_\ell' = 1, 0)$ or extending funds and late liquidation, $(r^{P\prime}, \mathbb{1}_\ell' = 1)$. The profit schedule for the bank $E\Pi'(\sigma_1', \sigma_2')$ is thus the same as the incumbent bank, although they will face different incentive constraints. Thus, if the competitor bank offers contracts (σ_1', σ_2'), its profits are

$$E\Pi'(\sigma_1', \sigma_2') = \begin{cases} R\ell & \text{if } (\sigma_1', \sigma_2') = (\mathbb{1}_\ell' = 1, 0) \\ r^{P\prime} + \ell & \text{if } (\sigma_1', \sigma_2') = (r^{P\prime}, \mathbb{1}_\ell' = 1) \\ p(b')b' + (1 - p(b'))\ell + Rr' & \text{if } (\sigma_1', \sigma_2') = (r', b') \neq 0 \end{cases} \tag{2}$$

conditional on the contract being accepted. Notice that, unlike the incumbent bank, the competitor bank has no original claims on the firm, D_0. Therefore, even if it can theoretically offer to liquidate the firm, this is a contract that will never be offered in equilibrium.

Firms. At the beginning of the game, the firm has a technology that allows it to obtain a revenue of $y_0 = p_0^{-\alpha}$, where p_0 is the probability of success or baseline risk. In period 1, the firm can produce using this inherited technology. In period 2, the expected revenue with this technology is $Ey_0 = p_0 y_0 = p_0^{1-\alpha}$ and, if it fails with probability $(1 - p_0)$, the firm gets zero. In other words, y_0 is the "business as usual" revenue stream.

In period 1, besides producing using its traditional technology (and get y_0), the firm can also invest in project that improves technology with different risk. Projects are chosen from a continuum and are indexed by their probability of success $p_i \in [0, 1]$. In case of success, each project induces an increase in revenue from $y_0 \equiv p_0^{-\alpha}$ to $y_1 = p_i^{-\alpha}$ in period 2. The parameter $\alpha \in [0, 1]$ captures the elasticity of revenue to the riskiness of the project. In this way, it is a measure of risk shifting: the safer is the project, the lower is the revenue in case of success. I assume that the firm can only carry on one investment project. The cost of investment, X, is the same for all projects p_i. Finally, I assume that the choice of project is not contractible: the firm borrows from the bank but cannot commit to invest following a particular risk profile.

The firm also has an exogenous, previously contracted level of debt, D_0, with a bank; the incumbent. This will affect banks' incentives but is already a sunk cost. If the firm refuses to pay the contracted debt with its incumbent, it can be liquidated and its assets seized.

The value of the firm at the beginning of the game is,

$$\max_{\mathbb{1}_I} E\pi = \max_{\mathbb{1}_I} \{y_0 - r - \mathbb{1}_I X + [\mathbb{1}_I EV_i(b) + (1 - \mathbb{1}_I)p_0(y_0 - b)]\} \quad (3)$$

Where y_0 is the level of output in period 1, $\mathbb{1}_I$ is an indicator function equal to one if the firm invests, r is the payment in period 1 to the bank, which may be positive (in which the bank takes funds) or negative (in which the bank extends funds). The term in square brackets represents profits in period 2 if the firm invests ($\mathbb{1}_I = 1$) or if it does not invest ($\mathbb{1}_I = 0$). Notice that r may be contracted with a different bank than the one that lent D_0. The value of the firm in period 2, if it decides to carry on project i, is

$$EV_i(b) = \max_i \left\{p_i(p_i^{-\alpha} - b)\right\}$$

where p_i is the probability of success of project i, b is repayment on new funds. The firm can choose to carry on a new project or remain its business-as-usual mode, obtaining an expected revenue of $p_0 y_0$. Note that EV_i has an inverse-U shape with a maximum at an intermediate efficient level of risk taking. The first order condition, conditional on the firm investing is

$$p^*(b) = \left(\frac{1-\alpha}{b}\right)^{1/\alpha} \quad (4)$$

which depends on b. Equation (4) shows how the firm's attitudes towards risk are shaped by the level of repayment expected by its bank. The reaction function presents risk shifting: the higher the debt, the lower the probability of success of the project chosen in equilibrium.

Incentive Constraints. Each agent seeks to maximize their own profits. However, there are several incentive constraints that should be taken into account when banks make offers and the firm decides whether to accept or reject such offers.

(A) IC for firm

First, the firm will accept any contract with weakly positive profits since otherwise it would be liquidated (in which case the firm gets zero). Thus, the value of the firm should satisfy

$$\max_{\mathbb{1}_I} \{y_0 - r + \mathbb{1}_I(EV_i(b) - X) + (1 - \mathbb{1}_I)p_0(y_0 - b)\} \geq 0 \tag{5}$$

In words, if the firm is not liquidated, it will only operate if profits are weakly positive, whether investing or keeping its business-as-usual revenue, y_0, choosing optimally whether to invest or not. If the firm is liquidated, it will get a value of zero.

(B) Feasibility

Secondly, the project needs to be feasible, in the sense that profits cannot be negative for any r. In period 1, this includes the payments for investment and to the bank. In period 2, this takes into account the repayment to the bank. If the firm decided to invest, its profits will be y_1, whereas if the firm stayed with its business as usual project, it will get y_0. Thus, feasibility entails the following set of constraints

$$\begin{cases} y_0 - r - \mathbb{1}_I X \geq 0 & t = 1 \\ y_1 - b \geq 0 & t = 2 \wedge \mathbb{1}_I = 1 \\ y_0 - b \geq 0 & t = 2 \wedge \mathbb{1}_I = 0 \end{cases} \tag{6}$$

(C) IC for Competitor Bank

Third, following Myers [1977] and Fama and Miller [1972], I assume debt seniority in repayment. This is stated as

Assumption 3.1. *Old debt is senior to new debt*

Assumption 3.1, is crucial for the results of the paper. The reasoning is that if it were not the case, the issuing of new claims could dilute expected repayment for the incumbent bank, b. If the firm becomes insolvent, the incumbent bank would have to share the recovery value with the competitor. Anticipating this behavior, all banks would refuse to lend in the first place. An equivalent statement is to say that the competitor bank will make an offer (r', b') only if the firm can make enough profits as to pay its original debt, D_0 [Kovrijnykh and Szentes, 2007]. The following constraint describes these contracts,

$$\max_{\mathbb{1}_I} \left\{ y_0 - r' + \mathbb{1}_I (EV_i(b') - X) + (1 - \mathbb{1}_I) p_0 (y_0 - b') \right\} \geq D_0 \tag{7}$$

(D) IC for Incumbent Bank

The incumbent bank will offer a contract (r, b) such that the expected repayment is at least as high as the opportunity cost of all the extended funds in the first period. If the competitor offers a contract that satisfies this constraint, it is also profitable for the incumbent bank to do the same. Without loss of generality, I assume that the incumbent is the one accepted when both banks make the same offer. Therefore, the following incentive compatibility will only apply in equilibrium to the incumbent bank when it decides to extend new funds,

$$p(b)b + (1 - p(b))\ell \geq Rr \tag{8}$$

where $p(b)$ is success of the project and ℓ is the liquidation value of the firm, which the bank will get in case of failure.

3.2 Equilibrium Analysis: Market Structure

Market structure depends on the inherited level of debt. For low levels of debt, the firm is solvent and the competitor bank is willing to make offers that push the incumbent to the competitive outcome. Both banks will make a competitive offer to finance investment. The incumbent bank will make an additional offer in which it requests the original debt to be repaid. On the other hand, if the inherited level of debt is large enough such that the firm is insolvent, then the incumbent bank becomes a monopolist. This is a situation of debt overhang, and in this case, the incumbent bank will maximize profits subject to the incentive constraints of the firm. The firm will choose a project from its full array of possibilities. This is summarized in Proposition 3.2.

Proposition 3.2. (Market Structure) *Assume the firm possesses a project that increases social surplus and increases its expected output. Then, there exists a threshold for the inherited level of debt, $\bar{D}(y_0)$, such that*

- *(competitive markets) For $D_0 < \bar{D}(y_0)$, both banks make offer $\hat{r} = -X$ and $\hat{b} = RX$. The incumbent bank makes an additional offer of $\hat{\hat{r}} = D_0$.*
- *(debt overhang) For $D_0 \geq \bar{D}(y_0) > y_0$, the incumbent bank maximizes Equation (1), subject to Equation (6), Equation (8), Equation (4) and Equation (8) if it extends new funds.*

The threshold for debt is a function on initial output, as the relevant measure is the debt to output ratio. In other words, to generate debt overhang, it is equivalent to shock the original level of output (decreasing it) or the original level of debt (increasing it). It is worth noting that I do not explain how debt arrives to that level and I will simply assume that this level is hit by a zero measure shock that pushes original debt beyond the threshold in Section 3.4.[6]

In the next sections I will analyze the properties of the equilibrium under both competitive and non competitive markets.

[6]Kovrijnykh and Szentes [2007] shows how this arises with probability one as a sequence of bad shocks. In their model, debt overhang happens with probability one, but the agent takes precautions to decrease its occurrence.

3.3 Equilibrium under Competition

When the firm has access to competitive lending, competition will push banks' profits towards zero. Both banks will offer $b = RX$, offer to extend funds $r = -X$, and the incumbent bank will also get D_0. In this section, I normalize D_0 to zero purely for easiness of exposition, as it does not distort any decision. I assume without loss of generality that the firm will choose its incumbent bank when presented with equal offers from both. The following proposition summarizes the reaction of the firm under competitive markets.

Proposition 3.3. *Under competitive markets as in Proposition 3.2, and $D_0 \leq \bar{D}(y_0)$, then the firm will implement project $\hat{p}$, given by*

$$\hat{p} = p(RX) = \left(\frac{1-\alpha}{RX}\right)^{1/\alpha} \tag{9}$$

for $RX \geq 1 - \alpha$

In other words, risk neutral banks will compete in prices providing loans until they make an offer at the opportunity cost of those funds. The firm will carry on the project given that it is preferable to the revenue stream from its business-as-usual technology. In doing so, it will take on an efficient level of risk, given by the reaction function of the firm. Firms repay their debt (normalized to zero), and request funds to do an investment project. They receive the competitive rate and take on an optimal level of risk.

Throughout the paper, I will focus on investment projects that are efficient. In this paper, a project is efficient if it is voluntarily carried on under competitive markets, and increases social surplus. The following proposition summarizes the set of investment projects for which this is true.

Proposition 3.4. *If $RX \in I_X \equiv \left[(1-\alpha), \max\{\ell, \left(\frac{\alpha p_0^\alpha}{\epsilon}\right)^{\frac{\alpha}{1-\alpha}}\}\right]$, then the project is efficient.*

The lower bound of the I_X ensures that probabilities are bounded by 1. The upper bounds ensure that, under perfect competition, firms prefer to do the project than their business as usual revenue, and that social surplus is increased. Thus, for all $RX \in I_X$, under competitive markets, social welfare is maximum and the project with an intermediate level of risk is enacted, according to Equation (9).

3.4 Equilibrium under Monopolistic Banks

Now suppose there is a zero measure shock to original debt, ξ, such that $D_0' = D_0 + \xi \geq \bar{D}$. Following Proposition 3.2, the bank will have monopoly power given that the firm is now insolvent. Modeling debt overhang in this way allows me to abstract from ex-ante precautionary motives. Note that a shock to $\bar{D}$ is equivalent to a shock to y_0, such that $y_0' = \epsilon y_0 = \epsilon p_0^{-\alpha}$ with $\epsilon \in [0, 1]$. The firm is financially distressed due to interest payments but still has positive operational revenues. The case of a firm with operational losses will be studied in Section 3.5.

According to Proposition 3.2, bank now faces a maximization problem subject to incentives constraints. In this case, it can act as a monopolist and extract more than the competitive rate. The bank's strategies are to extend funds (r, b) or to liquidate the firm (either early or late). That is, the set of strategies is $(\sigma_1, \sigma_2) \in \{(r, b), (\mathbb{1}_\ell = 1, 0), (r^P, \mathbb{1}_\ell = 1)\}$. It is important to make two remarks. First, initial debt D_0' is a sunk cost and will play no role beyond granting monopoly power. Second, even though (r, b) is a continuous decision, there will only be two cases to consider for the initial repayment: (a) $r^Z = r^P = \epsilon y_0$ and (b) $r^I = \epsilon y_0 - X$, because otherwise there are idle funds or not enough to invest, and thus will be suboptimal. The set of possible strategies is stated in Proposition 3.5.

Proposition 3.5. *Under debt overhang as in Proposition 3.2, the set of bank's strategies is $\Sigma = \{I, Z, L, ZP\}$, where*

1. *(I) Funding investment, in which case it sets $(r^I = \epsilon y_0 - X, b^I = \frac{\ell}{1-\alpha})$,*

2. *(Z) Zombie-lending, in which case it sets* $(r^Z = \epsilon y_0, b^Z = \epsilon y_0)$

3. *(L) Liquidating, in which case it sets* $(\mathbb{1}_\ell = 1, 0)$.

4. *(ZP) Partial Zombie-Lending, in which case it sets* $(r^P = \epsilon y_0, \mathbb{1}_\ell = 1)$.

In options (Z) the bank takes the full operational profits from the firm without lending for further investment. I interpret (Z) as zombie-lending. In strategy (ZP), the bank offers $(r^P, \mathbb{1}_\ell = 1)$. I interpret (ZP) as partial zombie-lending. In this case, the bank obtains the full operational profits of the firm in the first period, but liquidates the firm at the beginning of the second.

Notice that even if $r \geq 0$, this fits into the empirical counterpart used by the literature for zombie-lending of subsidized lending, even if revenue is positive. The reason is that when taking into account the roll-over costs, the firm's profits are indeed negative. The original debt, D_0, is a sunk cost at this point but made the firm insolvent. Thus, given that $D_0' \equiv \xi \geq r^Z = \epsilon y_0$ as per Proposition 3.2, then the firm is indeed receiving subsidized lending. In this section the firms under study will be financially distressed; i.e insolvent due to interest payments. The case of an operationally insolvent firm will be studied in the next section.

Figure 3 shows expected profits from the different possible strategies of the bank. The schedule for bank's profits of strategy (I) (funding investment) ,$E\Pi^I$,is non monotonous as the possibility of risk shifting decreases the incentives to enact the efficient project. In other words, larger repayments b increase the risk and thus decrease repayment. The schedule for bank's profits under zombie-lending strategy (Z), are given by $E\Pi^Z$. Given that the firm does not change its technology, there is no scope for risk shifting and the probability of success remains p_0. Profits increase monotonically until they are truncated due to feasibility at the maximum possible repayment. The profit schedule from liquidation, $E\Pi^L$, is a constant at $R\ell$ (i.e, early liquidation and using the riskless technology). When the bank uses a strategy of partial zombie-lending, $E\Pi^{ZP}$, it decides to take the firm's revenues as payment in the first period, and put it in the riskless technology. In the second period, it liquidates the firm. Thus, it foregoes the interest on

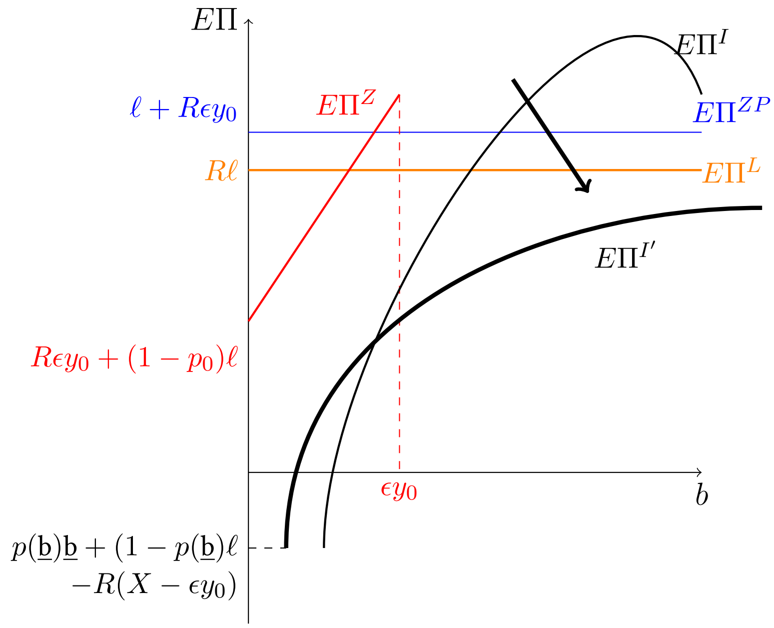

Figure 3: Profit functions for each strategy of the bank. The arrow shows the flattening effect on $E\Pi^I$ of increasing α.

liquidation, as this is done only in period 2.

Formally, the bank solves the following maximization problem,

$$
\begin{aligned}
\max_{\{Z,I,L,ZP\}\in\Sigma} & \left\{E\Pi^{I}, E\Pi^{Z}, E\Pi^{L}, E\Pi^{ZP}\right\} \quad \text{s.t} \\
(i) \quad & E\Pi^{Z}(r^{Z}, b^{Z}) = p_0 b^{Z} + (1-p_0)\ell + Rr^{Z} \geq 0 \\
(ii) \quad & E\Pi^{L}(\mathbb{1}_{\ell}=1, 0) = R\ell \geq 0 \\
(iii) \quad & E\Pi^{ZP}(r^{P}, \mathbb{1}_{\ell}=1) = Rr^{P} + \ell \geq 0 \\
(iv) \quad & E\Pi^{I}(r^{I}, b^{I}) = p(b^{I})b^{I} + (1-p(b^{I}))\ell + Rr^{I} \geq 0 \\
& \qquad \text{s.t} \quad p(b) = \left(\frac{1-\alpha}{b}\right)^{1/\alpha} \\
(v) \quad & \text{Feasibility (Eq. (6)).}
\end{aligned}
$$

Notice that the only constraint missing according to Proposition 3.2 is (D). Notice however that this is slack . In strategy (I), if $r \leq 0$ and $b \geq 0$ holds trivially. if $r \geq 0$ as in strategy (Z), then it is slack due to the non negativity of profit functions. The same constraint for the strategy (ZP) is implied by the maximization between $E\Pi^{ZP}$ and $E\Pi$.

When there is a debt overhang problem, the amount of money that the bank can obtain from the firm is not determined by the opportunity cost, but it maximizes the amount of money that can be extracted from a firm. Therefore, the bank can push the debt beyond the competitive level of debt. The higher pressure pushes the firm towards higher risk taking thus lowering social welfare. The following result follows from this proposition,

Lemma 3.6. *When firms are in debt overhang as in Proposition 3.2, $RX \in I_X$, and $\ell \geq (1-\alpha)^2$, social welfare is not higher than under competitive markets. Liquidation is socially inefficient.*

This Lemma states that, under debt overhang and in the presence of efficient projects, social welfare is lower. The restriction of $\ell \geq (1-\alpha)^2$ ensures that probability is well defined. Welfare is lower due to an unnecessary level of risk taking, unexploited investment opportunities or inefficient liquidation. Liquidation is inefficient because the firm is financially distressed, but its operational revenues are still positive.

A financially insolvent firm with a profitable project does not receive fresh funds, and instead receives help rolling over their original debt. Lack of solvency is ensured by the definition of debt overhang. I now find conditions under which the zombie-lending strategy will be chosen by the bank, even when there are profitable projects and liquidation is allowed as a strategy.

Theorem 3.7. (Zombie Firms) *Let $\Omega(p_0, \epsilon, \phi, \alpha, RX) \in [0,1]^4 \times I_X$ be the set of admissible parameters. Under debt overhang as in Proposition 3.2, when $\alpha \to 1$,*

- *If $\epsilon \leq p_0 \frac{R-1}{R-\phi R+\phi}$, bank chooses (L)*
- *if $\epsilon \in [p_0 \frac{R-1}{R-\phi R+\phi}, p_0 \frac{\phi}{1-\phi}]$, bank chooses (ZP)*
- *if $\epsilon \geq p_0 \frac{\phi}{1-\phi}$, bank chooses (Z).*

Theorem 3.7 states that when there is a high degree of risk shifting the bank may find it more profitable to keep the firm alive by rolling over its financial costs without investing even if the firm has a profitable project.

When the disruption to the firm is low (i.e, high ϵ) the bank will do pure zombie-lending, (Z). This is because the amount of profits to be extracted without new projects in the firm is large enough. In particular, it is large with respect to the disruption costs, $1-\phi$. If these disruption costs are intermediate, the bank will do partial zombie-lending. This means, the bank will prefer to keep the firm alive in the $t=1$ but to liquidate it in $t=2$. This relates to the interest rate, as higher interest rates are a force in favor of early liquidation. Higher ϕ and lower p_0 decreases the threshold for ϵ, and thus makes the space of parameters for zombies to arise larger. Higher ϕ, meaning that disruption costs are low, give more incentives to liquidate. In terms of the figure, increased disruption costs shift the liquidation profit function downwards.

The key of the theorem is that higher scope for risk shifting (larger α) flattens the investment curve -shown by the arrow to the thicker curve in Figure 3. This also increases the lower bound of b, $\underline{b} = 1 - \alpha$ such that the probability is well defined. The amount that can be extracted by the remaining strategies is unchanged. Strategy (L), or early liquidation $(\mathbb{1}_{\ell} = 1, 0)$ gives the bank $E\Pi^{L}$. Partial zombie lending (or late liquidation) yields $E\Pi^{ZP}$. Both $E\Pi^{ZP}$ and $E\Pi^{L}$ do not depend on b, and thus are flat. Thus, under a monopolistic behavior bank, increasing b decreases the profitability of lending for investment.

Given that there is a profitable project, the bank would like to extend funds in normal situations. However, if the bank decides to extend the funds for investment, the necessary repayment for this to be profitable would persuade the firm to increase the riskiness of the project to a point in which is not profitable anymore. Without funding investment, the bank can obtain all current output via zombie-lending or partial zombie-lending.

Notice that in this section we are focusing in firms in financial distress, but still viable given that its revenues are positive when not taking into accounts interest payments. Finally, it is relevant to stress the conceptual difference between zombie-lending and liquidation. Even though the bank takes all the revenue when it decides to zombie-lend or partial zombie-lend, this is a very different strategy from liquidating. The reason is that when the firm is liquidated, after paying disruption costs, it is possible to enact the (efficient) investment project instead of business-as-usual technology.

3.5 The Case of a Financially Distressed Firm with Operational Losses.

In the previous sections I analyzed the effect of a firm in debt overhang with positive operational revenues but financially distressed, i.e, with negative profits due to large interest payments. In this section, I will study a firm with operational losses while financially distressed. Financial distress is what grants the bank monopolistic power. In its absence, the firm with operational losses would still be able to gather funds to enact efficient projects.

I allow for operational losses in the model by including a fixed cost in the firm's profits, κ. The firm's profits in this case are given by

$$
\begin{array}{ll}
\max_{\mathbb{1}_I} E\pi & = \max_{\mathbb{1}_I}\{y_0 - r - \kappa + \mathbb{1}_I(EV_i(b) - X) + (1-\mathbb{1}_I)p_0(y_0 - b - \kappa)\} \\
EV_i & = \max_i \left\{p_i(p_i^{-\alpha} - b) - \kappa\right\}
\end{array}
\tag{10}
$$

Notice that firm's profits can be negative in the first period, but the value of the firm can still be positive under the implementation of a profitable project. Moreover, even if the firm is having operational losses in the first period, the liquidation value may be positive given that the firm may be profitable if the investment project is carried out. As can be seen from Equation 10, the structure of the problems is mostly unaffected by this changes, but two features need to be adapted: the set of efficient investment projects and the liquidation value. The following proposition characterizes the new sets.

Proposition 3.8. *Let* $(p_0, \epsilon, \phi, \alpha, \kappa) \in [0,1]^4 \times R_+$ *and firm's profits are given by Equation (10). If the firm is in debt overhang as in Proposition 3.2, when* $\alpha \to 1$,

1. $\ell \to \max\{0, \phi\left(\frac{\epsilon}{p_0} + 1 - 2\kappa\right)\}$.
2. *Liquidation is socially inefficient if* $\quad 2\kappa \leq \frac{\epsilon}{p_0} + 1$.
3. *The firm makes losses if* $\kappa \geq \frac{\epsilon}{p_0}$.
4. *Investment is efficient if* $\quad b \in I'_X \equiv (0, \ell]$.

The liquidation value will be positive if the losses in the first period are compensated by profits in the second period when the company invests efficiently. An efficient investment project will be enacted as long as the investment cost is lower than the potential recovery value for the bank. Finally, if the firm has this profitable project, then it is inefficient to liquidate it, given that the firm is still viable.

A natural question is whether we can observe zombie lending in this situation. The following proposition characterizes the values of the profit functions for the bank when liquidation is socially inefficient. (I) is not an equilibrium strategy because the firm will increase its risk thus making it a non profitable investment. Zombie lending (Z) is dominated by partial zombie lending, (ZP), since it entails covering less losses. Since (ZP) implies no change in the technology of the firm, it will continue to have operational losses in both periods. Thus, (ZP) will yield negative profits for the bank. Since the profits of liquidating the firm, (L) is bounded below by zero, there will never be zombie-lending under operational losses. This is stated in Theorem 3.9.

Theorem 3.9. (No zombie lending with operational losses). *Let* $\tilde{\Omega}(p_0, \epsilon, \phi, \alpha, \kappa, RX) \in [0,1]^4 \times R_+ \times I'_X$. *When there is debt overhang as in Proposition 3.2 and the firm has operational losses (*$\kappa \geq \epsilon/p_0$*) then the bank chooses* (L)*, both if it is efficient or inefficient.*

The Theorem states that when a bank will choose to liquidate the firm if it has operational losses. This liquidation may be efficient or inefficient. Liquidation will be efficient when the net present value of the firm is negative, meaning that it will sustain operational losses even when enacting their best projects. Liquidation will be inefficient when a firm has a project that will increase profits. However, the bank lacks the technology to monitor this project, and cannot induce it with a debt contract due to moral hazard. Thus, a company that is viable is disrupted, paying social costs ϕ.

3.6 The Case of a Financially Distressed Bank

Stylized facts point to the fact that undercapitalized banks are more likely to keep the firms in their portfolios as zombies. To explore this possibility, consider a simple modification to the game, shown in Figure 4. Assume now that the bank has capital of A in its balance sheet and there are minimum capital requirements, K. If book capital is lower than capital requirements, $A < K$, the bank goes bankrupt and obtains a value of ν. Moreover, assume that there is a shock, ψ to

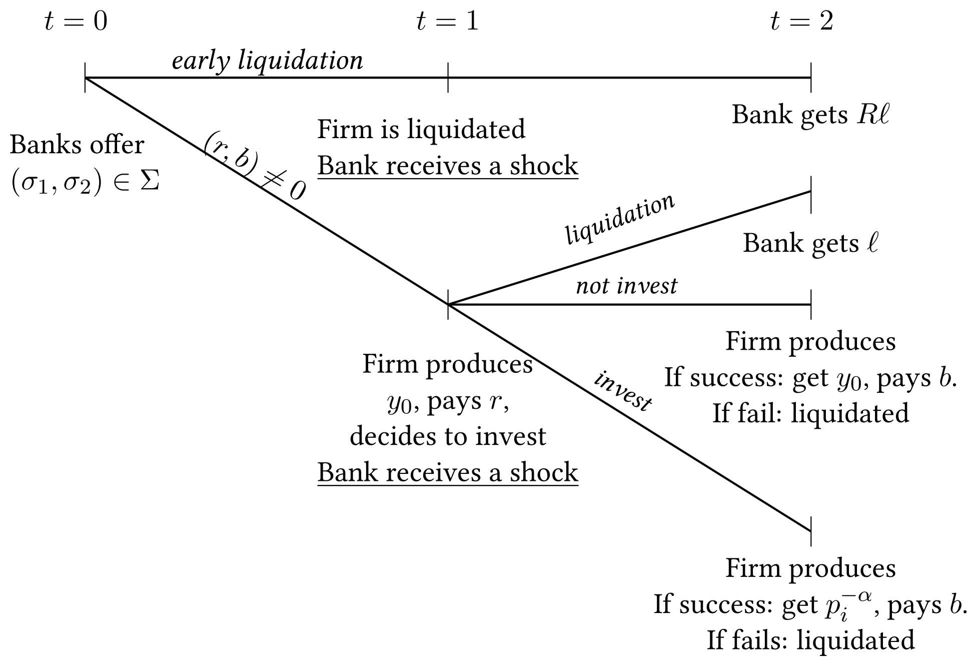

Figure 4: Timing of the model with a financially distressed bank.

bank capital in period 1 such that $A' = A - \psi$. The shock ψ is distributed according to a smooth accumulated distribution with support on the positive domain, $G(\cdot) \in [0, \infty]$. This can push the bank beyond the minimum capital requirements and thus bankruptcy risk arises. If the bank liquidates the firm in period zero, it would acknowledge the loss of assets in its books. I assume that the bank will be capitalized at the end of the game if it decides to keep the firm afloat instead of liquidating in period zero.[7]

Therefore, bank's profits if it decides to liquidate in period 1 are

$$E\Pi^L = R\ell \left(1 - G\left(A - K\right)\right) + G(A - K)\nu \tag{11}$$

[7]This is a tractable way to include preference for delaying liquidation in the bank. The modelling choice is discussed in Section 5.

Where A is declared book capital. If the bank does not recognize its loss, assets will be the original A_0. By liquidating the firm, the bank writes down the value of the assets to $A_0' \leq A_0$. In this case, if the bank liquidates the firm in period 0, it gets instantly the liquidation value, ℓ. This will earn $R\ell$ in period 2, since it can be allocated to the alternative investment. However, expected profits are now weighed by the probability of bank survival; i.e, the probability that the shock is smaller than the difference between declared assets and minimum capital requirements. Moreover, if that happens, the bank obtains ν.

Figure 5, shows the effect of acknowledging losses, shifting downwards the expected revenue function from liquidating. The schedule of funding investment is unaffected, as well as the schedule for zombie-lending.

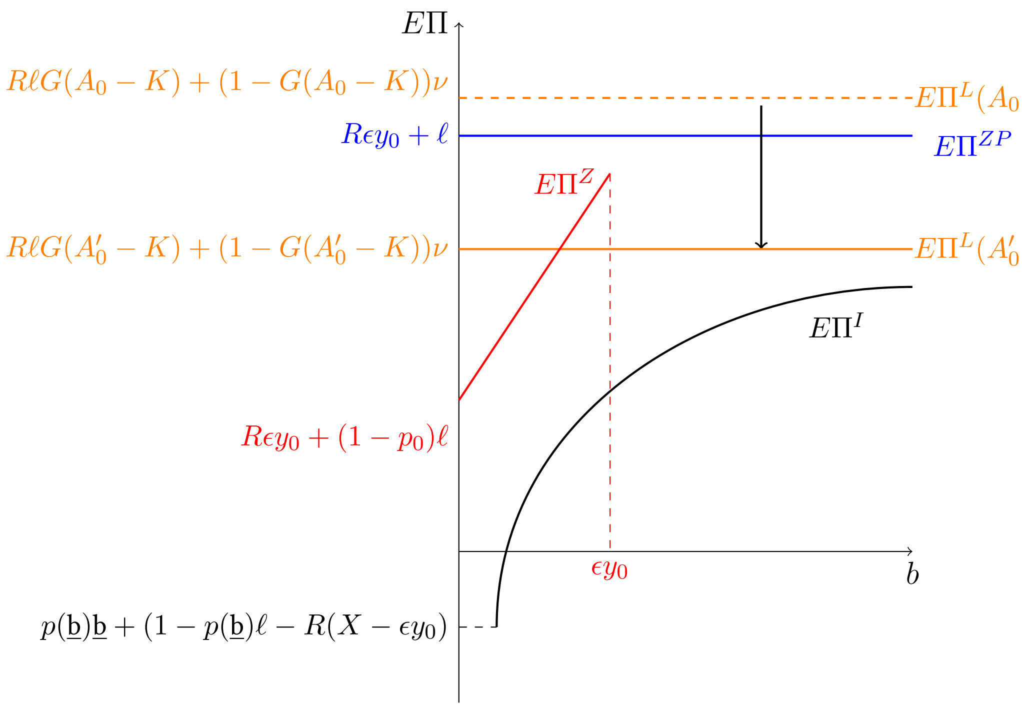

Figure 5: Profit functions of a financially distressed bank when $\alpha \to 1$. The arrow shows the effects of lowering bank's book assets.

Now I turn to show how zombies arise in this modified game for both cases.

Financially Distressed Firm With Positive Revenue

The next theorem states that the conditions for a financially distressed firm under a monopolistic bank are laxer when a bank is undercapitalized. This is because the strategy of liquidation, which was an equilibrium strategy in the previous section, is no longer an attractive option for the bank if it entails a high risk of going bankrupt. The space of parameters for which (ZP) is an equilibrium is enlarged. In particular, the lower the value of the bank when bankrupt, ν, the less likely is that the bank will liquidate. In particular if $\nu \leq \phi$, the bank always chooses (ZP) over (L). High interest rates in this case make it more likely to have zombies via (ZP), given that it makes it more attractive to put the revenue in the risk free technology, and there is not counterweight from the benefit of liquidating early, as the bank is at financial risk.

Theorem 3.10. (Zombie Firms with an undercapitalized banks and positive revenue). *Let $\nu \leq R\ell$ and $\tilde{\Omega}(p_0, \epsilon, \phi, \alpha, RX) \in [0,1]^4 \times I'_X$. Under Debt overhang as in Proposition 3.2, when $\alpha \to 1$ an undercapitalized bank ($A \to K$) prefers to do (Z) if $\epsilon \geq p_0 \frac{\phi}{1-\phi}$, (ZP) if $\epsilon \in [\frac{\nu-\phi}{R+\phi} p_0, p_0 \frac{\phi}{1-\phi}]$ and (L) otherwise*

Financially Distressed Firm with operational losses

According to Theorem 3.9, if presented with a firm with positive operational revenue, the bank will choose (Z) or (ZP), in which the bank does not fund investment and keeps the firm alive for one or two periods. If, on the other hand, the firm has operational losses, the revenues for the bank are negative in the first period, and they will continue to be negative if the efficient project is not enacted. Thus, the bank will sustain losses by following (Z) or (ZP). Strategy (ZP) dominates strategy (Z), given that it allows the bank to liquidate in period 2 without facing any risk, thus limiting its losses to zero. The bank's profits from liquidating are given by Equation (11). Thus, if $\nu \geq 0$, liquidating the firm will give a return of at most zero and this will dominate the remaining strategies. Thus, the only possible way to observe partial zombie-lending is when the bank is financially distressed (i.e, low assets) and the value of its own bankruptcy, ν, is negative. This is stated in Theorem 3.11.

Theorem 3.11. (Zombie Firms with an undercapitalized banks and negative revenue). *Let* $\tilde{\Omega}(p_0, \epsilon, \phi, \alpha, \kappa, RX) \in [0,1]^4 \times R_+ \times I'_X$. *Under Debt overhang as in Proposition 3.2, an undercapitalized bank* $(A \to K)$*, with a firm with operational losses* $(\epsilon < \kappa p_0)$*, when* $\alpha \to 1$ *there exists a threshold* $\bar{\nu} < 0$ *such that*

- *if* $\nu \geq \bar{\nu}$ *the bank prefers* (L)*,*
- *if* $\nu \leq \bar{\nu}$. *the bank prefers* (ZP).

The theorem states that for sufficiently low value of bankruptcy of the bank, the bank covers the firm's losses in the first period, until it is able to liquidate the firm in the second period when its capitalized. In other words, the bank trades a small loss in terms of covering the losses of the bank in order to reduce a larger loss in its own bankruptcy. This is inefficient for two reasons. First, there exists a project that can increase social surplus that is not enacted. Secondly, the theorem shows that this is independent of any other parameters, so it is possible that liquidation is inefficient (i.e, with positive net present value).

4 Policy Implications

Now I discuss several policies implications that arise from the model.

Debt Haircuts. The underlying reason why zombies arise is the presence of moral hazard by the firms that limit the willingness of the bank to extend new funds. The bank is unwilling to grant further funds $(r < 0)$ because the amount that it would have to require on the next period to make it profitable (b) is so large that it is not compensated by the increase in risk. The bank, in turn, prefers to extract all the available funds from the firm for itself. If capitalized, it does not choose to liquidate in order to avoid the disruption costs. In this context, a debt haircut limits the capacity of the bank to extract funds from the firm and aligns the incentives for efficient risk taking. Provided the haircut is large enough, the firms can regain access to the competitive market. Risk is reduced and expected social surplus increases. This allows the possibility of compensating banks making everyone better off. Notice that it is not optimal for banks to privately offer a debt haircut, since they cannot ensure they will profit from the surplus of the new project. Moreover, they would also loose their monopolistic gains of extracting all income from the firm. Next proposition states this result. When the monopolistic bank decides on inefficient liquidation, the debt haircut allows the firm to be relieved from this locked in relationship and obtain funds in the competitive market to enact its efficient project. In this way, the economy as a whole can avoid the disruption costs and surplus is increased.

Proposition 4.1. (Debt Haircuts.) *In the presence of zombies as in Theorem 3.7 or in inefficient liquidation in Theorem 3.9, there exists a debt haircut $\zeta \in [0,1]$ on original debt D_0, and lump-sum transfers τ such that at the new level of debt $D_0' = (1-\zeta)D_0$, both the bank and the firm are better off.*

Notice that the proposition cannot make any statement regarding an undercapitalized bank, since this policy could increase bankruptcy risk for this agent. Thus, there is a need for recapitalization, which will be discussed at the end of this section.

Monetary Policy. Lower interest rates decrease the opportunity cost of funds, thus making banks more prone to lend for investment. However, this also makes liquidation less desirable, partially neutralizing the effect. The next proposition states that there is a set of parameters such that reducing the interest rate has no effect on investment as the as the fundamental incentive problem of risk shifting under debt overhang is too strong.

Proposition 4.2. (Monetary Policy)

1. *Capitalized bank*

   (a) *Financially distressed firm (Theorem 3.7)*

    i. *If (Z) was the equilibrium, will remain as such for any $R \geq 1$,*

    ii. *Lowering R increases the space of parameters for which (ZP) is an equilibrium, and never increases investment.*

  (b) *Financially and operationally distressed firm (Theorem 3.9)*

    i. *Inefficient liquidation remains an equilibrium for any R.*

2. *Undercapitalized bank*

  (a) *Financially distressed firms (Theorem 3.10).*

    i. *When (Z) is the equilibrium, it remains as such for any R.*

    ii. *The space for which (ZP) is chosen decreases as R increases, and is smaller the lower is ν.*

  (b) *Financially and operationally distressed firm (Theorem 3.11)*

    i. *If it chooses (ZP), it remains an equilibrium for any R.*

The first part of the Proposition states that if banks are capitalized and the firm has positive operational revenue, lowering the interest rate does not affect the incentives to lend for investment. This is because the firm, overburdened with debt, would not use the funds efficiently. Moreover, lowering the interest rate enlarges the space of parameters for which partial zombie lending is an equilibrium, given that it decreases the incentives to liquidate early. When there are no zombies but the bank inefficiently liquidates firms, lowering the interest rate does not change the equilibrium strategy to investing, which is socially desirable. Finally, when there are firms receiving zombie lending that are financially and operationally distressed, this is because the bank is too close to its own bankruptcy and thus will choose to do partial zombie-lending irrespective of the the interest rate. However, when an undercapitalized bank is paired with a financially distressed firm with positive revenues, the interest rate plays the expected role. Lower rates make survival lending less profitable, and thus liquidation relatively more attractive. However, this effect gets smaller as the bankruptcy value of the bank is smaller.

Bank recapitalization. When zombies arise with capitalized banks, obviously recapitalizing banks has no effect. The only trade off for banks is between risk shifting and the revenues they can obtain. Recapitalization could potentially have an effect under the conditions in Theorems 3.10 and 3.11, when zombies arise partly due to the distorted liquidation decision. However, it may not be enough because it does not fix the trade-off between zombie-lending and funding investment given that risk shifting and debt overhang are still present. Thus, at most, recapitalization will induce liquidation, either efficient (when the net present value of the firm is negative) or inefficient. Inefficient liquidation only happens when firms with a profitable project are financially stressed but do not have operational losses. Thus, disruption costs constitute a social loss. In other words, the sudden increase in debt turned the firm no longer viable, and thus would benefit from renegotiation. This is stated in Proposition 4.3.

Proposition 4.3. (Bank Capitalization)

1. *When the bank is capitalized, if firms have positive operational revenue (Theorem 3.7) or negative operational revenues (Theorem 3.9), equilibrium strategy does not change.*
2. *When the bank is not capitalized,*
   (a) *Small capitalizations can be ineffective*

   (b) *If the firm is financially distressed (Theorem 3.10) if (Z) is the equilibrium, the strategy does not change. If the bank chooses (ZP), capitalization is ineffective if $\epsilon \geq \phi p_0 \frac{R-1}{R-R\phi+\phi}$.*

   (c) *When the firm has operational losses (Theorem 3.11), capitalization induces (L), efficient or inefficient*

Notice finally that in the condition of Theorem 4.3, 2b, the RHS increases as R

increases. Thus, lower interest rates increases the range of parameters for which recapitalization is ineffective.

5 Discussion and Conclusion

Directing credit flows to distressed firms is a phenomenon that has been observed in several countries after the burst of a bubble. This paper explains why zombie-lending arises and continues to thrive, even when renegotiation is possible, efficient projects are available and banks are capitalized.

The model captures a fundamental conflict between lender and borrower in a double-decked incentive problem. The main drivers of this result are that the borrower is locked-in a lending relationship with its incumbent bank and that it has access to a risk shifting technology. For this, it is assumed that the shock is measure-zero, and thus completely unexpected. This modeling choice is motivated by the empirical fact that zombie firms spiked after the burst of bubbles. In any case, Nosal and Ordoñez [2016] show how uncertainty in policy responses may lead to strategic restraint and allow banks to self discipline even when shock is expected. Once the firm is in debt overhang, the incumbent bank can extract more funds than socially optimal from the firm. Firms are not be willing to use the fresh funds efficiently, and the bank anticipates this behavior. Previous explanations for delays in restructuring rely on a type of "gambling for resurrection" either due to distressed banks [Bruche and Llobet, 2013], strategic negotiation [Admati and Perry, 1987], or to uncertainty [Vilanova, 2004]. In my model, financially distressed firms are viable and have efficient projects, but these are not enacted when debt is large even when the bank has funds. Moreover, the incentives of the actors do not allow the renegotiation process to solve the problem.

The mechanism present in the model has policy implications in stark contrast to previous papers, and in line with several stylized facts (Section 2). Debt haircuts are shown to be necessary to restore investment. Fukuda and Nakamura [2011] show empirically that debt relief and capital reduction were important for the recovery in Japan. Unlike in Gorton and Kahn [2000], where banks sometimes accept to soften the initial credit terms in order to reduce the distressed firms' risk shifting incentives; in my model the required repayment in the next period is too large as to be profitable to fund investment.

The model explains why bank capitalization is not sufficient to restart investment, at least when this capitalization is not large enough. Strengthening banks is insufficient if insolvency regimes are hostile to the reorganization of indebted firms, as it only attacks one side of the incentive problem. The model generates a typology of firms that may receive subsidized lending and the conditions under which a bank may decide to pursue this strategy. Giannetti and Simonov [2013] presents evidence that this was exactly the case in Japan.

The model also explains the ineffectiveness of monetary policy to reignite investment without the need of the zero lower bound. The model could be extended to infinite periods allowing for firm entry. In this case, if the bank does not internalize the effect of zombie-lending, it may deter potential entrants by increasing the value of resources. This externality would cause policies to have larger impact on welfare.

Several assumptions are essential to obtain the results. First, in the model, debt contracts are non-contingent. Investment is non-contractible and therefore the offers take into account the scope for risk shifting. The result would not hold if the bank could make offers conditional on specific projects. However, as long as the bank's monitoring technology is not perfect, the results should hold to a certain degree.

Second, bank opacity is necessary for the case of undercapitalized banks. Otherwise, investors would price the asset loss in the market value of the bank.[8] It could be argued that regulators should have superior information and thus could put sanctions directly. Hellwig et al. [2012], Bruche and Llobet [2013] and Hoshi [2006] discuss the issue of regulatory forbearance, which is assumed away in this paper. The main results hold even even with no regulatory forbearance and capi-

[8] BIS [2011], Flannery et al. [2004] and Huizinga and Laeven [2012] provide evidence.

talized banks, under stronger conditions. Undercapitalized banks are modeled as being transitorily fragile, represented by a shock in the intermediate period. The main reason for this choice is tractability. However, in practice, banks normally smooth out losses over many periods using provisions.[9] This modeling choice effectively abstracts from the possibility of a firm taking debt speculating with bank bankruptcy.

Third, I assume that the firm has debt with only one bank.[10] I abstract from potential conflicts of interest between lenders to highlight the debt overhang channel. Notwithstanding, syndicated loans have been growing increasingly more common, which may cause a large number of creditors to behave collusively.[11] Moreover, there are some institutional solutions that facilitate the financing of distressed firms, such as debt-equity swaps or DIP financing [Kahl, 2002]. This can be understood within the strategy of liquidation by including transaction or monitoring costs. Alternatively, it is possible to frame these strategies as to requiring some acknowledgement of losses, and thus addressed within the context of a a financially distressed bank.

Fourth, I assume that voluntary bankruptcy is not allowed or, equivalently, that is not preferred to remaining a zombie. Otherwise, an arrangement as Chapter 11 should restore efficiency.[12] In a way, my model assumes market incompleteness for renegotiation and/or a loss of reputation associated with bankruptcy [Ordonez, 2018].[13] Large cross-country differences in these aspects may explain the relative prevalence of zombies found in papers such as Andrews and Petroulakis [2019].

[9] Rajan [1994] offers a model with reputational concerns that generates this behavior.
[10] Noe and Wang [2000] and Bolton et al. [1993] focus on the role of many creditors.
[11] See Gadanecz [2004] and Esty and Megginson [2003].
[12] See for example Annabi et al. [2012] and Franks and Torous [1994].
[13] Evidence of this can be found in Semadeni et al. [2008] and Gilson and Vetsuypens [1993].

6 Proofs

Proof of . Proposition 3.2. First we show that If D_0 is lower than $\bar{D}$, both banks offer $\hat{r} = -X$ and $\hat{b} = RX$. Assume it is not an equilibrium offer. If $\hat{r} \neq -X$, there is either not sufficient funds for investing or idle funds, so it is not an equilibrium. If the incumbent bank offers $\hat{b}' < \hat{b}$, then $\Pi < 0$, thus contradicting bank's incentive compatibility. If the incumbent bank offers $\hat{b}' > \hat{b}$, then the competitor can offer $\hat{b}'' = \hat{b}' - \varepsilon$, for arbitrarily small ε and have its offer accepted. Since there are enough funds to invest and the project is profitable, firm reacts using Equation (4) thus doing project $p(\hat{b})$. Lastly, if the incumbent bank offers to liquidate, $\mathbb{1}_\ell = 1$, then the firm gets zero profits. Then, the competitor can make an offer $(-X, \hat{b} - \varepsilon)$ such that $\Pi > 0$ and $\pi > 0$. Same logic applies to the competitor making offers. Thus, it is not an equilibrium.

If $D_0 > \bar{D} \equiv \gamma \{(y_0 - r - X(I) + \max_i\{EV_i(b), p_0 y_0\}\} \geq y_0$, where $\gamma > 1$. The claim is the bank will offer a strategy $(\sigma_1, \sigma_2) \in \{(\tilde{r}, \tilde{b}), (\tilde{r}', \mathbb{1}_\ell^I), (\mathbb{1}_\ell^I, 0)\}$ where $(\tilde{r}, \tilde{b}) \in \mathbb{R}$, chosen as to maximize profits as in Equation (1) subject to Equation (4), (6) and (8). Notice that if Equation (6) holds and the project is efficient (i.e, $EV_i \geq 0$), then equation (5) is slack. Now, suppose it is not an equilibrium. Then the incumbent bank can increase profits by offering $\tilde{b} + \epsilon$, which contradicts the maximization. The competitor bank can never make an offer below $\tilde{b}$, since Equation 7 never holds. Since $\pi(b) > 0$ for all b, then the firm is always strictly better accepting any offer from the incumbent bank. Similar argument holds for liquidation decision. □

Proof of . Proposition 3.3. By Bertrand-competition, both banks will offer $\hat{b} = RX$. By replacing in Equation 4, we obtain the desired result. $RX \geq 1 - \alpha$ ensures probabilities are well defined. □

Proof of . Proposition 3.4. The indirect profit function for the firm is given by

$$E\pi(b) = \left(\frac{1-\alpha}{b}\right)^{\frac{1-\alpha}{\alpha}} - \left(\frac{1-\alpha}{b}\right)^{1/\alpha} b$$

Outside value of not investing is $p_0 y_0$. By replacing $\hat{b} = RX$ and operating, we find that if $RX \in I_X$ investment is preferred. For probability to be well defined we need that $RX \geq 1-\alpha$. Maximizing social surplus is equivalent to maximizing welfare,

$$\max_b W(p(b)) = p^{1-\alpha} - b + (1-p)\ell \to \frac{\partial W}{\partial b} = \frac{\partial p}{\partial b}(b-\ell) - 1$$

Substituting by the functional form of $p'(b)$, it is immediate that if $b \leq \ell$ then derivative is positive. $\square$

Proof of . Proposition 3.5. First I show that each of these strategies is an equilibrium. Then, I show that these are the only possible strategies.

1. First we show that (r^I, b^I) is an equilibrium strategy if the bank wants to fund investment. $r^I = \epsilon y_0 - X$ is an equilibrium given that any $r' \neq r^I$ is either not enough to fund investment or leaves idle resources. To find b^I, replace the reaction function of the firm in the problem of the bank and maximize,

   $$\Pi = \left(\left(\frac{1-\alpha}{b}\right)^{1/\alpha} b + (1 - \left(\frac{1-\alpha}{b}\right)^{1/\alpha})\ell + Rr \right) = (1-\alpha)^{(1-\alpha)} b^{\frac{-1}{\alpha}}(b-\ell)$$

   The maximization yields,

   $$-(1-\alpha)^{\frac{1}{\alpha}} \frac{b^{\frac{-1}{\alpha}-1}}{\alpha}(b-\ell) + (1-\alpha)^{\frac{1}{\alpha}} b^{\frac{-1}{\alpha}} = 0 \to b^M = \frac{\ell}{1-\alpha}$$

2. If the bank decides to zombie-lend, it sets $(r^Z = \epsilon y_0, b^Z = \epsilon y_0)$. Assume, on the contrary, that $r^Z > \epsilon y_0$, then the firm makes negative profits, thus violating feasibility. Assume $r^Z < \epsilon y_0$. Since bank's profits are increasing in r, $r' = r^Z + \varepsilon$ with arbitrary small ε, is feasible and yields higher profits.

3. Liquidating is trivially a strategy the bank finds it profitable to liquidate.

4. Setting $r^P = \epsilon y_0$ in the first period, there are no idle funds. Liquidation in

the second period is trivially a strategy. Moreover, notice that $(r^P, \mathbb{1}_\ell = 1)$ dominates $(r^I, \mathbb{1}_\ell = 1)$.

Say the bank offers r'' such that $r^Z > r'' > r^I$, then there are either not enough funds to invest (in which case it cannot extract b^I) or idle funds. Thus, only possible equilibrium strategies are $\{(r^Z, b^Z), (r^I, b^I), (\mathbb{1}_\ell, 0), (r^P, \mathbb{1}_\ell)\}$. $\square$

Proof of . Lemma 3.6. If the does not lend enough for investment, a project that increases welfare is not enacted. Liquidation is inefficient since implies disruption costs ϕ and missed efficient projects. From Proposition 3.3, all that remains to show now is that $\hat{p} = p(RX) \geq p^I(b^I)$. Since $RX \geq (1-\alpha)$, and if $(1-\alpha)^2 \geq \ell$, this is true. Under Proposition 3.4, the project at $\hat{p}$ is efficient and increases welfare. Thus, welfare is lower. $\square$

Proof of . Theorem 3.7.
Relevant Sets. Let $\Omega = \left\{(p_0, \alpha, \phi, \epsilon, RX) \in [0,1]^4 \times I_X, \right\} \subset \mathbb{R}^5$, such that

$$
\begin{aligned}
&y_0 = p_0^{-\alpha} \in (0, \alpha]\,, \\
&\ell = \phi\left(\epsilon y_0 + \alpha\left(1-\alpha\right)^{\frac{1-\alpha}{\alpha}}\left(RX\right)^{-\frac{1-\alpha}{\alpha}}\right) \geq (1-\alpha)^2 \\
&RX \in I_X \equiv \left[(1-\alpha), \max\left\{\ell, (1-\alpha)\left(\frac{\alpha p_0^\alpha}{\epsilon}\right)^{\frac{\alpha}{1-\alpha}}\right\}\right]
\end{aligned}
$$

Ensuring that probabilities are well defined, and efficiency of investment. First, notice that

$$
\ell = \phi\left(\epsilon y_0 + \alpha\left(1-\alpha\right)^{\frac{1-\alpha}{\alpha}}\left(RX\right)^{-\frac{1-\alpha}{\alpha}}\right) = \phi\left(\epsilon p_0^{-\alpha} + \alpha\left(\frac{1-\alpha}{RX}\right)^{\frac{1-\alpha}{\alpha}}\right)
$$

Which, since $\lim_{\alpha\to 1}\alpha\left(\frac{1-\alpha}{RX}\right)^{\frac{1-\alpha}{\alpha}}=1$, and $\lim_{\alpha\to 1}p_0^{-\alpha}=\frac{1}{p_0}$, converges to

$$\lim_{\alpha\to 1}\ell=\phi\left(1+\frac{\epsilon}{p_0}\right) \tag{12}$$

Second, since $\lim_{\alpha\to 1}(1-\alpha)\left(\frac{\alpha p_0^{\alpha}}{\epsilon}\right)^{\frac{\alpha}{1-\alpha}}=+\infty$ and $\lim_{\alpha\to 1}\ell=\phi\left(1+\frac{\epsilon}{p_0}\right)$, then $\lim_{\alpha\to 1}I_X=\left[0,\phi\left(1+\frac{\epsilon}{p_0}\right)\right]$.

Part A. We now find conditions such that $E\Pi^Z\geq E\Pi^I$. Bank will prefer to offer $r^Z=\epsilon y_0$ and $b^Z=\epsilon y_0$ instead of $r^I=\epsilon y_0-X$ and $b^I=\frac{\ell}{1-\alpha}$ if

$$R\epsilon y_0+\epsilon p_0 y_0+(1-p_0)\ell\geq R\epsilon y_0-RX+p(b)b+(1-p(b))\ell$$

Replacing and simplifying,

$$RX+\epsilon p_0^{1-\alpha}-p_0\ell-\alpha\left(1-\alpha\right)^{\frac{2-\alpha}{\alpha}}\ell^{-\frac{1-\alpha}{\alpha}}\geq 0 \tag{13}$$

Taking limits in each term

$$\begin{aligned}
&\lim_{\alpha\to 1}\epsilon p_0^{1-\alpha}=\epsilon\\
&\lim_{\alpha\to 1}p_0\ell=\phi(p_0+\epsilon)\\
&\lim_{\alpha\to 1}\alpha\left(1-\alpha\right)^{\frac{2-\alpha}{\alpha}}\ell^{-\frac{1-\alpha}{\alpha}}=0
\end{aligned}$$

Then, $E\Pi^Z\geq E\Pi^I$ if

$$RX+\epsilon-(p_0+\epsilon)\phi\geq 0$$

which holds for nonempty $\Omega'\in\Omega$.

Part B. Now we find conditions under which $E\Pi^Z\geq E\Pi^L$. This is true if

$$R\epsilon y_0+\epsilon p_0 y_0+(1-p_0)\ell\geq R\ell$$

Replacing and taking limits, when $\alpha \to 1$

$$\epsilon(R + p_0) \geq (R - 1 + p_0)\phi(p_0 + \epsilon)$$

Remark #1: Notice that if $\epsilon \geq p_0 \frac{\phi}{1-\phi}$, the LHS grows faster than the RHS when changing R. Thus, if the condition holds for $R = 1$, will hold for any $R \geq 1$. Setting $R = 1$,

$$\epsilon(1 + p_0) \geq p_0^2\phi + \phi p_0 \epsilon \to p_0^2(-\phi) + p_0(1 - \phi)\epsilon + \epsilon \geq 0$$

The last equation is a quadratic equation in p_0, and is straightforward to show that is positive for any value of the parameters. Thus, $E\Pi^Z \geq E\Pi^L$ unconditionally.

Part C. Now we find conditions under which the bank prefers $E\Pi^Z$ to $E\Pi^{ZP}$. This is true if

$$E\Pi^{ZP} = \ell + Ry_0 \leq Ry_0 + p_0 y_0 + (1 - p_0)\ell = E\Pi^Z$$

Simplifying and taking limits $\alpha \to 1$,

$$\epsilon \geq \frac{\phi}{1 - \phi} p_0 \tag{14}$$

Notice that this condition contains is exactly the same as in Remark #1.

Part D. From Part C, we know that if $\epsilon \leq \frac{\phi}{1-\phi} p_0$ then $E\Pi^{ZP} \geq E\Pi^Z$. Thus, we need to find conditions for $E\Pi^{ZP} \geq E\Pi^L$ and $E\Pi^{ZP} \geq E\Pi^I$. By taking limits, we find that $E\Pi^{ZP} \geq E\Pi^I$ when $\alpha \to 1$, unconditionally. Finally,

$$E\Pi^{ZP} = R\epsilon y_0 + \ell \geq R\epsilon y_0 + \epsilon p_0 y_0 + (1 - p_0)\ell = E\Pi^L$$

Reworking the conditions and taking limits,

$$\epsilon \geq \frac{R - 1}{R - \phi R + \phi} \phi p_0$$

which is lower than $\frac{p_0\phi}{1-\phi}$. Combining with the Part C, we establish that for $\epsilon \in [p_0\frac{R-1}{R-\phi R+\phi}, p_0\frac{\phi}{1-\phi}]$, $E\Pi^{ZP}$ is chosen in equilibrium.

□

Proof of . Proposition 3.8.

1. The liquidation value in this case is the profits in the first period plus the future stream of profits of enacting a project that is profitable,

$$\begin{aligned}\lim_{\alpha\to 1} \ell &= \lim_{\alpha\to 1} \max\left\{0, \phi\left(\epsilon y_0 + \alpha\left(1-\alpha\right)^{\frac{1-\alpha}{\alpha}}\left(RX\right)^{-\frac{1-\alpha}{\alpha}} - 2\kappa\right)\right\} \\ &= \max\left\{0, \phi\left(\frac{\epsilon}{p_0} + 1 - 2\kappa\right)\right\}\end{aligned}$$

2. The net present value of the firm is the term in brackets, and thus if $\frac{\epsilon}{p_0} + 1 - 2\kappa \le 0$ liquidation is efficient.

3. Profits in first period are $\frac{\epsilon}{p_0} - \kappa > 0 \to \kappa \ge \frac{\epsilon}{p_0}$.

4. The indirect profit function for the project is

$$E\pi(b) = \left(\frac{1-\alpha}{b}\right)^{\frac{1-\alpha}{\alpha}} - \left(\frac{1-\alpha}{b}\right)^{1/\alpha} b - \kappa$$

   and the project will be efficient for the firm if $E\pi(b) \ge 0$. This is true if $b \ge (1-\alpha)\kappa^{\frac{-\alpha}{1-\alpha}} \to_{\alpha\to 1} 0$. For society, the project will be efficient if it increases social surplus under competitive markets,

$$\max_b W(p(b)) = p^{1-\alpha} - b + (1-p)\ell - \kappa$$

   Taking the first order condition, project increases social welfare if $b \ge \ell$ since $p' > 0$ and $b \ge 0$. For the probabilities to be well defined the repayment $b \ge (1-\alpha)$. Thus, $b \in \left[\min\left\{1-\alpha, (1-\alpha)\kappa^{\frac{-\alpha}{1-\alpha}},\right\}, \ell\right] \to_{\alpha\to 1} (0, \ell]$.

□

Proof of . Theorem 3.9.

The profit functions for each strategy are

$$\lim_{\alpha\to 1} E\Pi^Z = \lim_{\alpha\to 1}\{R(\epsilon y_0 - p_0\kappa) + \epsilon p_0 y_0 + (1-p_0)\ell - \kappa\} \tag{15}$$

$$= R\left(\frac{\epsilon}{p_0} - \kappa\right) + \epsilon - \kappa + (1-p_0)\phi\left(\frac{\epsilon}{p_0} + 1 - 2\kappa\right) \tag{16}$$

$$\lim_{\alpha\to 1} E\Pi^I = \lim_{\alpha\to 1}\{R(\epsilon y_0 - \kappa) - RX + p(b)b + (1-p(b))\ell - \kappa\} \tag{17}$$

$$= R\left(\frac{\epsilon}{p_0} - \kappa\right) - RX - \kappa + \phi\left(\frac{\epsilon}{p_0} + 1 - 2\kappa\right) \tag{18}$$

$$\lim_{\alpha\to 1} E\Pi^{ZP} = \lim_{\alpha\to 1}\{R(\epsilon y_0 - \kappa) + \ell\} \tag{19}$$

$$= R\left(\frac{\epsilon}{p_0} - \kappa\right) + \phi\left(\frac{\epsilon}{p_0} + 1 - 2\kappa\right) \tag{20}$$

$$\lim_{\alpha\to 1} E\Pi^Z = \lim_{\alpha\to 1} R\ell = R\phi\left(\frac{\epsilon}{p_0} + 1 - 2\kappa\right) \tag{21}$$

Since the firm has operational losses, $\epsilon \leq \kappa p_0 \to \epsilon \leq \kappa$. Then $E\Pi^Z \leq E\Pi^L$, since it includes two negative numbers and since $p_0 \leq 1$, a term smaller than ℓ. $E\Pi^I \leq E\Pi^L$ since the extra terms in $E\Pi^I$ are negative.Thus, can only be partial zombie lending if (ZP) is greater than (L). Since $\epsilon \leq \kappa p_0$, and $R \geq 1$, $E\Pi^{ZP} \leq \ell \leq R\ell = E\Pi^L$. Notice that this is independent of the value of ℓ, since it can be positive (inefficient liquidation) or zero (efficient liquidation). □

Proof of . Theorem 3.10. Notice that profit functions for $E\Pi^Z, E\Pi^L, E\Pi^{ZP}$ given by Equations (16), (18) and (20) remain as in previous proof. $E\Pi^L$, instead of Equation (16) is now $R\ell(1 - G(A-K)) + G(A-K)\nu$. Say now the bank has very little capital, so $A \to K$, since is continuously non increasing in A, $G(\cdot) \to 1$. Thus, $E\Pi^L \to \nu = 0 \leq R\ell$. From Theorem 3.7, we know that bank prefers $E\Pi^Z$ to $E\Pi^L = R\ell$ and $E\Pi^I$ unconditionally. Moreover, $E\Pi^Z \geq E\Pi^{ZP}$ if $\epsilon \geq p_0\frac{\phi}{1-\phi}$. Finally, since $E\Pi^{ZP} \geq \nu \iff \epsilon \geq \frac{\nu-\phi}{R+\phi}p_0$. □

Proof of . Theorem 3.11. Notice that profit functions for $E\Pi^Z, E\Pi^L, E\Pi^{ZP}$ given by Equations (16), (18) and (20) remain as in previous proof. $E\Pi^L$, instead of Equation (16) is now $R\ell(1-G(A-K))+G(A-K)\nu$. Comparing Equations (21) and (20), it is clear (Z) is dominated by (ZP) since $\epsilon \le \kappa p_0$. Moreover, from Equations (21) and (18), (ZP) dominates (I). When $A \to K$, $E\Pi^L \to \bar{\nu} < 0$. Thus, for $\bar{\nu} = E\Pi^{ZP}$, banks prefer to do (ZP) instead of (L), and viceversa if condition is not met. □

Proof of . Proposition 4.1. Welfare is maximized under competitive markets from Proposition 3.3. From Proposition 3.2, market structure depends on the level of debt. Suppose debt is $D_0 = \xi$. A haircut $\zeta > 0$ turns debt into $D_0' = (1-\zeta)\xi < \bar{D}(y_0)$. From Proposition 3.6, welfare is non decreasing. Increased surplus can be redistributed in a lump sum fashion using transferences τ such that both agents are better off. □

Proof of . Proposition 4.2. .

1. Capitalized bank
   (a) From Theorem 3.7)
      i. Bank prefers to do (Z) if $\epsilon \ge p_0\frac{\phi}{1-\phi}$, at any level of R.
      ii. Bank prefers to do (ZP) if $\epsilon \in [p_0\phi\frac{R-1}{R-\phi R+\phi}, p_0\frac{\phi}{1-\phi}]$. Notice the lower bound of the set increases in R, and the upper bound is unaffected. Moreover, when $R \to 1$ the set becomes $[0, p_0\frac{\phi}{1-\phi}]$.
   (b) Conditions in Theorem 3.9), are unaffected by R.
2. Undercapitalized bank
   (a) From Theorem 3.10, (ZP) if $\epsilon \in [\frac{\nu-\phi}{R+\phi}p_0, p_0\frac{\phi}{1-\phi}]$. Increasing R increases the lower bound.
   (b) R does not affect any of the conditions for Theorem 3.11.

□

Proof of . Proposition 4.3. Under Theorem 3.7, and 3.9, the level of assets of banks is irrelevant, so it is trivially true. Under Theorem 3.10, the only strategy that is affected is (L). Given the continuity and monotonicity of G, there is a level of bank assets $\bar{\bar{A}} \geq A_0$ such that $R\ell G(\bar{\bar{A}} - K) \sim R\ell$. Thus, we revert to results from Theorem 3.7. Finally, from Theoreom 3.11, we know that $E^{ZP} \leq \ell$. Increasing A increases $E\Pi^L$ from $\nu 0$ to ℓ. All other strategies remain dominated, and given continuity, (L) will be chosen. All that remaisn to show is that small capitalizations can be ineffective. Results follows from continuity and assuming any of the conditions for the chosen equilibrium strategies is slack.

□

References

Viral V Acharya, Tim Eisert, Christian Eufinger, and Christian W Hirsch. Whatever it takes: The real effects of unconventional monetary policy. *Available at SSRN 2740338*, 2016.

Anat R Admati and Motty Perry. Strategic delay in bargaining. *The Review of Economic Studies*, 54(3):345–364, 1987.

Alan G Ahearne and Naoki Shinada. Zombie firms and economic stagnation in japan. *International Economics and Economic Policy*, 2(4):363–381, 2005.

Heitor Almeida, Murillo Campello, and Michael S Weisbach. Corporate financial and investment policies when future financing is not frictionless. *Journal of Corporate Finance*, 17(3):675–693, 2011.

Dan Andrews and Filippos Petroulakis. Breaking the shackles: zombie firms, weak banks and depressed restructuring in europe. 2019.

Amira Annabi, Michèle Breton, and Pascal François. Resolution of financial distress under chapter 11. *Journal of Economic Dynamics and Control*, 36(12):1867–1887, 2012.

Nicolas Aragon. Zombie firms and interbank linkeages: Evidence from spain. 2018.

Yan Bai and Jing Zhang. Duration of sovereign debt renegotiation. *Journal of international Economics*, 86(2):252–268, 2012.

Amir Barnea, Robert A Haugen, and Lemma W Senbet. A rationale for debt maturity structure and call provisions in the agency theoretic framework. *the Journal of Finance*, 35(5):1223–1234, 1980.

Erik Berglof and Gerard Roland. Soft budget constraints and banking in transition economies. *Journal of Comparative Economics*, 26(1):18–40, 1998.

BIS. Bank of international settlements, 82nd annual report. (Annual Report):26, 2011.

Patrick Bolton, David S Scharfstein, et al. Optimal debt structure with multiple creditors. Technical report, European Science Foundation Network in Financial Markets, c/o CEPR, 77 âĂę, 1993.

Max Bruche and Gerard Llobet. Preventing zombie lending. *The Review of Financial Studies*, 27(3):923–956, 2013.

Ricardo J Caballero, Takeo Hoshi, and Anil K Kashyap. Zombie lending and depressed restructuring in japan. Technical report, " American Economic Review, 2008.

Hui Chen and Gustavo Manso. Macroeconomic risk and debt overhang. *Review of Corporate Finance Studies*, 6(1):1–38, 2017.

Abe De Jong and Ronald Van Dijk. Determinants of leverage and agency problems: A regression approach with survey data. *The European Journal of Finance*, 13(6):565–593, 2007.

Ryan A Decker, John C Haltiwanger, Ron S Jarmin, and Javier Miranda. Changing business dynamism and productivity: Shocks vs. responsiveness. Technical report, National Bureau of Economic Research, 2018.

Douglas W Diamond and Raghuram G Rajan. Fear of fire sales and the credit freeze. Technical report, National Bureau of Economic Research, 2009.

Gauti Eggertsson, Andrea Ferrero, and Andrea Raffo. Can structural reforms help europe? *Journal of Monetary Economics*, 61:2–22, 2014.

Assaf Eisdorfer. Empirical evidence of risk shifting in financially distressed firms. *The Journal of Finance*, 63(2):609–637, 2008.

Benjamin C Esty and William L Megginson. Creditor rights, enforcement, and debt ownership structure: Evidence from the global syndicated loan market. *Journal of Financial and Quantitative Analysis*, 38(1):37–60, 2003.

Eugene F Fama and Merton H Miller. *The theory of finance*. Holt Rinehart & Winston, 1972.

Giovanni Favara, Erwan Morellec, Enrique Schroth, and Philip Valta. Debt enforcement, investment, and risk taking across countries. *Journal of Financial Economics*, 123(1): 22–41, 2017.

Mark J Flannery, Simon H Kwan, and Mahendrarajah Nimalendran. Market evidence on the opaqueness of banking firms' assets. *Journal of Financial Economics*, 71(3):419–460, 2004.

Julian R Franks and Walter N Torous. A comparison of financial recontracting in distressed exchanges and chapter 11 reorganizations. *Journal of financial economics*, 35 (3):349–370, 1994.

Pascal Frantz and Norvald Instefjord. Debt overhang and non-distressed debt restructuring. *Journal of Financial Intermediation*, 37:75–88, 2019.

Shin-ichi Fukuda and Jun-ichi Nakamura. Why "did zombie firms" recover in japan? *The world economy*, 34(7):1124–1137, 2011.

Blaise Gadanecz. The syndicated loan market: structure, development and implications. *BIS Quarterly Review, December*, 2004.

Mariassunta Giannetti and Andrei Simonov. On the real effects of bank bailouts: Micro evidence from japan. *American Economic Journal: Macroeconomics*, 5(1):135–67, 2013.

Erik P Gilje. Do firms engage in risk-shifting? empirical evidence. *The Review of Financial Studies*, 29(11):2925–2954, 2016.

Stuart C Gilson and Michael R Vetsuypens. Ceo compensation in financially distressed firms: An empirical analysis. *The Journal of Finance*, 48(2):425–458, 1993.

Gita Gopinath, Şebnem Kalemli-Özcan, Loukas Karabarbounis, and Carolina Villegas-Sanchez. Capital allocation and productivity in south europe. *The Quarterly Journal of Economics*, 132(4):1915–1967, 2017.

Gary Gorton and James Kahn. The design of bank loan contracts. *The Review of Financial Studies*, 13(2):331–364, 2000.

Oliver Hart and John Moore. Default and renegotiation: A dynamic model of debt. *The Quarterly Journal of Economics*, 113(1):1–41, 1998.

Oliver D Hart and Jean Tirole. Contract renegotiation and coasian dynamics. *The Review of Economic Studies*, 55(4):509–540, 1988.

Martin F Hellwig. A model of borrowing and lending with bankruptcy. *Econometrica: Journal of the Econometric Society*, pages 1879–1906, 1977.

Martin F Hellwig, André Sapir, Marco Pagano, Viral V Acharya, Leszek Balcerowicz, Arnoud Boot, Markus K Brunnermeier, Claudia Buch, Ieke van den Burg, Charles Calomiris, et al. Forbearance, resolution and deposit insurance. Technical report, Reports of the Advisory Scientific Committee, 2012.

Christopher A Hennessy. Tobin's q, debt overhang, and investment. *The Journal of Finance*, 59(4):1717–1742, 2004.

Takeo Hoshi. Economics of the living dead. *The Japanese Economic Review*, 57(1):30–49, 2006.

Harry Huizinga and Luc Laeven. Bank valuation and accounting discretion during a financial crisis. *Journal of Financial Economics*, 106(3):614–634, 2012.

Michael C Jensen and William H Meckling. Theory of the firm: Managerial behavior, agency costs and ownership structure. *Journal of financial economics*, 3(4):305–360, 1976.

Matthias Kahl. Economic distress, financial distress, and dynamic liquidation. *The Journal of Finance*, 57(1):135–168, 2002.

Sebnem Kalemli-Ozcan, Luc Laeven, and David Moreno. Debt overhang, rollover risk, and corporate investment: Evidence from the european crisis. Technical report, National Bureau of Economic Research, 2018.

Natalia Kovrijnykh and Balázs Szentes. Equilibrium default cycles. *Journal of Political Economy*, 115(3):403–446, 2007.

Paul R Krugman, Kathryn M Dominquez, and Kenneth Rogoff. It's baaack: Japan's slump and the return of the liquidity trap. *Brookings Papers on Economic Activity*, 1998(2): 137–205, 1998.

Gustavo Manso. Investment reversibility and agency cost of debt. *Econometrica*, 76(2): 437–442, 2008.

Nathalie Moyen. How big is the debt overhang problem? *Journal of Economic Dynamics and Control*, 31(2):433–472, 2007.

Stewart C Myers. Determinants of corporate borrowing. *Journal of financial economics*, 5(2):147–175, 1977.

Thomas H Noe and Jun Wang. Strategic debt restructuring. *The Review of Financial Studies*, 13(4):985–1015, 2000.

Jaromir B Nosal and Guillermo Ordoñez. Uncertainty as commitment. *Journal of Monetary Economics*, 80:124–140, 2016.

Maurice Obstfeld and Romain Duval. Tight monetary policy is not the answer to weak productivity growth. *Voxeu. com. http://voxeu. org/article/tight-monetary-policy-not-answer-weak-productivity-growth*, 2018.

Filippo Occhino and Andrea Pescatori. Debt overhang in a business cycle model. *European Economic Review*, 73:58–84, 2015.

Guillermo Ordonez. Confidence banking and strategic default. *Journal of Monetary Economics*, 100:101–113, 2018.

Grzegorz Pawlina. Underinvestment, capital structure and strategic debt restructuring. *Journal of Corporate Finance*, 16(5):679–702, 2010.

Joe Peek and Eric S Rosengren. Unnatural selection: Perverse incentives and the misallocation of credit in japan. *The American Economic Review*, 95(4):1144–1166, 2005.

Rohan Pitchford and Mark LJ Wright. Holdouts in sovereign debt restructuring: A theory of negotiation in a weak contractual environment. *The Review of Economic Studies*, 79 (2):812–837, 2012.

Raghuram G Rajan. Why bank credit policies fluctuate: A theory and some evidence. *The Quarterly Journal of Economics*, 109(2):399–441, 1994.

Fabiano Schivardi, Enrico Sette, and Guido Tabellini. Credit misallocation during the european financial crisis. 2017.

Matthew Semadeni, Albert A Cannella Jr, Donald R Fraser, and D Scott Lee. Fight or flight: Managing stigma in executive careers. *Strategic Management Journal*, 29(5):557–567, 2008.

Sheridan Titman and Sergey Tsyplakov. A dynamic model of optimal capital structure. *Review of Finance*, 11(3):401–451, 2007.

Laurent Vilanova. Bank seniority and corporate debt restructuring. In *EFA 2004 Maastricht Meetings Paper*, number 2880, 2004.